Pure Mathematics Algebra & Analytic Solid Geometry

GUIDE ANSWERS

Question Bank
& Practice Exams

3rd Sec
2022

EL-MOASSER

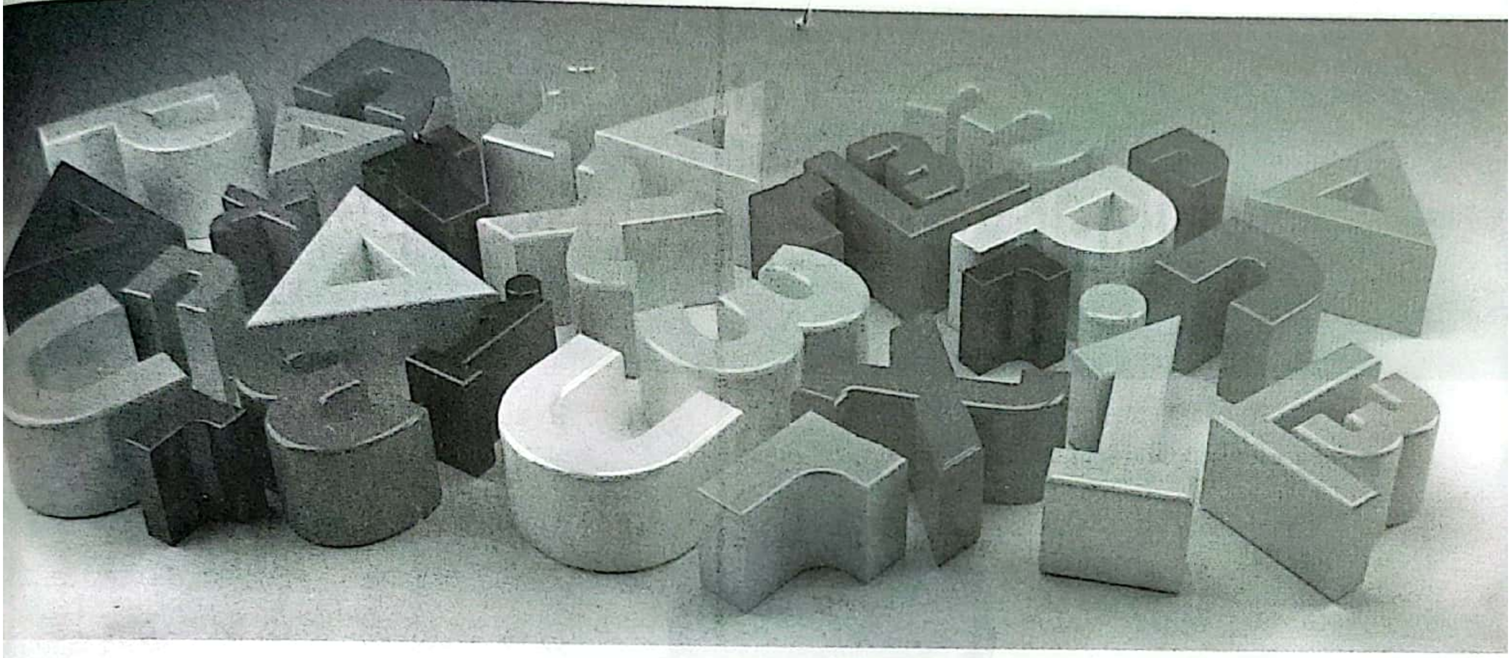
By a group of supervisors

EL-MOASSER

By a group of supervisors

PURE MATHEMATICS

Algebra & Analytic Solid Geometry



Question Bank
& Practice Exams

Guide Answers

GPS

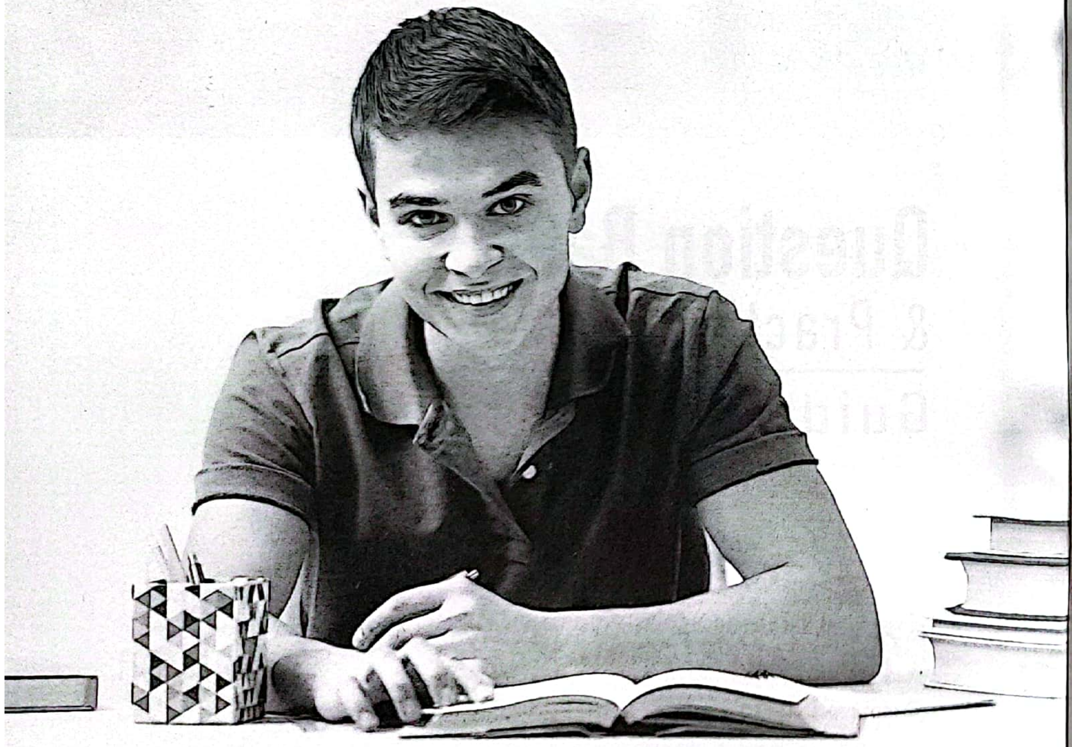
AL TALABA BOOKSTORE
For printing, publication & distribution
El Faggala - Cairo - Egypt
Tel.: 02/ 259 340 12 - 259 377 91
e-mail: info@elmoasserbooks.com
15014 www.elmoasserbooks.com

3rd Sec

Answers

of

Multiple Choice Question Bank



Answers of First : Question Bank in Algebra

First — Applications on permutations and combinations

1 (c)	2 (c)	3 (b)	4 (a)	5 (c)
6 (b)	7 (c)	8 (a)	9 (c)	10 (b)
11 (a)	12 (a)	13 (b)	14 (d)	15 (d)
16 (c)	17 (c)	18 (a)	19 (d)	20 (c)
21 (d)	22 (c)	23 (d)	24 (d)	25 (a)
26 (d)	27 (c)	28 (a)	29 (b)	30 (c)
31 (d)	32 (b)	33 (d)	34 (a)	35 (a)
36 (d)	37 (c)	38 (b)	39 (b)	40 (a)
41 (a)	42 (c)	43 (a)	44 (a)	45 (c)
46 (c)	47 (b)	48 (b)	49 (a)	
50 First : (a)	Second : (d)	Third : (c)		51 (c)
52 (a)	53 (a)	54 (a)	55 (a)	56 (b)
57 (a)	58 (c)	59 (d)	60 (d)	

Second — Questions on permutations and combinations rules

1 (b)	2 (c)	3 (a)	4 (b)	5 (b)
6 (d)	7 (b)	8 (c)	9 (c)	10 (b)
11 (a)	12 (c)	13 (d)	14 (d)	15 (c)
16 (b)	17 (b)	18 (b)	19 (c)	20 (c)
21 (b)	22 (b)	23 (b)	24 (c)	25 (c)
26 (c)	27 (d)	28 (c)	29 (d)	30 (c)
31 (a)	32 (b)	33 (d)	34 (a)	35 (d)
36 (b)	37 (c)	38 (b)	39 (d)	40 (b)
41 (a)	42 (c)	43 (a)	44 (c)	45 (c)
46 (d)	47 (c)	48 (c)	49 (a)	50 (b)
51 (c)	52 (d)	53 (b)	54 (d)	55 (c)
56 (d)	57 (c)	58 (d)	59 (b)	60 (b)
61 (a)	62 (d)	63 (a)	64 (a)	65 (c)
66 (a)	67 (c)	68 (d)	69 (c)	70 (d)
71 (a)	72 (a)	73 (b)	74 (c)	75 (c)
76 (d)	77 (a)	78 (b)	79 (b)	80 (d)
81 (b)	82 (d)	83 (c)	84 (d)	85 (a)
86 (a)	87 (b)	88 (d)	89 (b)	90 (d)
91 (c)	92 (c)	93 (d)	94 (a)	95 (c)
96 (d)	97 (d)	98 (d)	99 (c)	100 (c)
101 (d)	102 (a)	103 (d)	104 (c)	105 (d)
106 (b)	107 (d)	108 (c)	109 (d)	110 (d)
111 (c)	112 (a)	113 (c)	114 (c)	115 (a)
116 (a)	117 (d)			

Third — Questions on the binomial theorem

1 (d)	2 (c)	3 (a)	4 (a)	5 (c)
6 (b)	7 (b)	8 (a)	9 (c)	10 (a)
11 (a)	12 (b)	13 (d)	14 (b)	15 (d)
16 (b)	17 (b)	18 (b)	19 (a)	20 (c)
21 (a)	22 (c)	23 (c)	24 (d)	25 (a)
26 (b)	27 (b)	28 (a)	29 (b)	30 (c)
31 (b)	32 (a)	33 (a)	34 (b)	35 (a)
36 (c)	37 (c)	38 (d)	39 (c)	40 (a)
41 (d)	42 (a)	43 (b)	44 (a)	45 (c)
46 (a)	47 (a)	48 (c)	49 (b)	50 (b)
51 (c)	52 (b)	53 (a)	54 (c)	55 (a)
56 (b)	57 (b)	58 (a)	59 (c)	60 (d)
61 (b)	62 (b)	63 (b)	64 (c)	65 (c)
66 (c)	67 (c)	68 (b)	69 (c)	70 (b)
71 (a)	72 (c)	73 (d)	74 (c)	75 (d)
76 (d)	77 (b)	78 (a)	79 (c)	80 (b)
81 (d)	82 (d)	83 (c)	84 (c)	85 (c)
86 (d)	87 (b)	88 (a)	89 (d)	90 (b)
91 (d)	92 (d)	93 (c)	94 (a)	95 (c)
96 (c)	97 (c)	98 (d)	99 (a)	100 (c)

101 ⓑ 102 ⓓ 103 ⓐ 104 ⓒ 105 ⓒ
106 ⓐ 107 ⓒ 108 ⓑ 109 ⓒ 110 ⓒ
111 ⓒ 112 First : ⓑ Second : ⓓ 113 ⓐ
114 ⓒ 115 ⓓ 116 ⓐ 117 ⓒ 118 ⓒ
119 ⓑ 120 ⓐ 121 ⓐ 122 ⓐ 123 ⓒ
124 ⓓ

Fourth Questions on complex numbers

1

1 ⓐ 2 ⓓ 3 ⓓ 4 ⓓ 5 ⓐ
6 ⓒ 7 ⓒ 8 ⓐ 9 ⓒ 10 ⓐ
11 ⓐ 12 ⓐ 13 ⓓ 14 ⓒ 15 ⓓ
16 ⓑ 17 ⓓ

2

1 ⓒ 2 ⓐ 3 ⓒ 4 ⓒ 5 ⓑ
6 ⓑ 7 ⓑ 8 ⓑ 9 ⓐ 10 ⓒ
11 ⓑ 12 ⓐ 13 ⓑ 14 ⓑ 15 ⓒ
16 ⓐ 17 ⓑ 18 ⓒ 19 ⓑ 20 ⓓ
21 ⓓ 22 ⓒ
23 First : ⓑ Second : ⓐ Third : ⓑ 24 ⓑ
25 ⓑ 26 ⓐ 27 ⓒ 28 ⓐ 29 ⓓ
30 ⓐ 31 ⓒ 32 ⓑ 33 ⓓ
34 First : ⓓ Second : ⓐ 35 ⓓ 36 ⓒ
37 ⓑ 38 ⓑ 39 ⓑ 40 ⓒ 41 ⓒ
42 ⓐ 43 ⓐ 44 ⓒ 45 ⓓ 46 ⓒ
47 ⓐ 48 ⓑ 49 ⓒ 50 ⓒ 51 ⓑ
52 ⓐ 53 ⓑ 54 ⓓ 55 ⓒ 56 ⓐ
57 ⓐ 58 ⓓ 59 ⓓ 60 ⓒ 61 ⓐ
62 ⓓ 63 ⓓ 64 ⓒ 65 ⓓ 66 ⓓ
67 ⓒ 68 ⓓ 69 ⓒ 70 ⓓ 71 ⓐ
72 ⓐ 73 ⓒ 74 ⓒ 75 ⓒ 76 ⓐ
77 ⓓ 78 ⓑ 79 ⓐ 80 ⓒ 81 ⓐ
82 ⓒ 83 ⓐ 84 ⓑ 85 ⓒ 86 ⓓ
87 ⓒ 88 ⓑ 89 First : ⓐ Second : ⓓ
90 ⓑ 91 ⓒ 92 ⓒ 93 ⓓ 94 ⓒ
95 ⓓ 96 ⓑ 97 ⓑ 98 ⓑ 99 ⓒ
100 ⓑ 101 ⓑ 102 ⓐ 103 ⓒ 104 ⓒ
105 ⓓ 106 ⓓ 107 ⓐ
108 First : ⓒ Second : ⓓ 109 ⓒ
110 ⓓ 111 ⓒ 112 ⓒ 113 ⓒ 114 ⓑ
115 ⓒ 116 ⓑ 117 ⓐ 118 ⓐ 119 ⓑ
120 ⓑ 121 ⓑ 122 ⓓ 123 ⓑ 124 ⓑ
125 ⓒ 126 ⓐ 127 ⓐ 128 ⓐ 129 ⓐ
130 ⓐ 131 ⓐ 132 ⓓ 133 ⓑ 134 ⓐ
135 ⓐ

3

1 ⓑ 2 ⓓ 3 ⓐ 4 ⓑ 5 ⓐ
6 ⓑ 7 ⓑ 8 ⓒ 9 ⓑ 10 ⓒ
11 ⓒ 12 ⓒ 13 ⓑ 14 ⓓ 15 ⓒ
16 ⓓ 17 ⓒ 18 ⓐ 19 ⓑ 20 ⓐ
21 ⓒ 22 ⓒ 23 ⓒ 24 ⓓ 25 ⓑ
26 ⓒ 27 ⓑ 28 ⓒ 29 ⓓ 30 ⓒ
31 ⓑ 32 ⓒ 33 ⓐ

4

1 ⓒ 2 ⓒ 3 ⓒ 4 ⓓ 5 ⓐ
6 ⓑ 7 ⓐ 8 ⓓ 9 ⓓ 10 ⓓ
11 ⓐ 12 ⓑ 13 ⓒ 14 ⓒ 15 ⓑ
16 ⓑ 17 ⓑ 18 ⓒ 19 ⓐ 20 ⓓ
21 ⓒ 22 ⓓ 23 ⓐ 24 ⓒ 25 ⓐ
26 ⓑ 27 ⓓ 28 ⓓ 29 ⓐ 30 ⓐ
31 ⓓ 32 ⓓ 33 ⓒ 34 ⓑ 35 ⓐ
36 ⓒ 37 ⓐ 38 ⓑ 39 ⓒ 40 ⓑ
41 ⓑ 42 ⓐ 43 ⓑ 44 ⓑ 45 ⓑ
46 ⓓ 47 ⓒ 48 ⓑ 49 ⓑ 50 ⓑ
51 ⓐ 52 ⓓ 53 ⓓ 54 ⓑ 55 ⓐ
56 ⓑ 57 ⓐ 58 ⓐ 59 ⓒ 60 ⓒ
61 ⓒ 62 ⓐ 63 ⓐ 64 ⓐ 65 ⓑ

66 ⓐ 67 ⓓ 68 ⓓ 69 ⓑ 70 ⓐ
71 ⓐ 72 ⓑ 73 ⓐ 74 ⓓ 75 ⓒ

Fifth Questions on determinants

1 ⓒ 2 ⓓ 3 ⓑ 4 ⓐ 5 ⓓ
6 ⓑ 7 ⓐ 8 ⓓ 9 ⓓ 10 ⓐ
11 ⓒ 12 ⓑ 13 ⓑ 14 ⓐ 15 ⓒ
16 ⓐ 17 ⓐ 18 ⓐ 19 ⓑ 20 ⓑ
21 ⓓ 22 ⓓ 23 ⓒ 24 ⓐ 25 ⓑ
26 ⓒ 27 ⓑ 28 ⓐ 29 ⓓ 30 ⓓ
31 ⓓ 32 ⓑ 33 ⓑ 34 ⓒ 35 ⓒ
36 ⓓ 37 ⓒ 38 ⓓ 39 ⓒ 40 ⓑ
41 ⓐ 42 ⓒ 43 ⓐ 44 ⓑ 45 ⓒ
46 ⓒ 47 ⓓ 48 ⓓ 49 ⓐ 50 ⓐ
51 ⓑ 52 ⓐ 53 ⓑ 54 ⓑ 55 ⓐ
56 ⓒ 57 ⓐ 58 ⓓ 59 ⓓ 60 ⓐ
61 ⓒ 62 ⓑ 63 ⓑ 64 ⓒ 65 ⓑ
66 ⓒ 67 ⓓ 68 ⓐ 69 ⓓ 70 ⓐ
71 ⓓ 72 ⓓ 73 ⓐ 74 ⓓ 75 ⓐ
76 ⓑ 77 ⓒ 78 ⓓ 79 ⓑ 80 ⓑ
81 ⓒ 82 ⓐ 83 ⓐ 84 ⓑ 85 ⓓ
86 ⓐ 87 ⓒ 88 ⓑ 89 ⓓ 90 ⓐ
91 ⓓ 92 ⓐ 93 ⓑ

Sixth Questions on matrices and solving system of linear equations

1 ⓑ 2 ⓓ 3 ⓓ 4 ⓑ 5 ⓒ
6 ⓒ 7 ⓓ 8 ⓐ 9 ⓒ 10 ⓐ
11 ⓐ 12 ⓓ 13 ⓓ 14 ⓐ 15 ⓐ
16 ⓒ 17 ⓒ 18 ⓐ 19 ⓒ 20 ⓐ
21 ⓒ 22 ⓒ 23 ⓑ 24 ⓑ 25 ⓒ
26 ⓓ 27 ⓓ 28 ⓑ 29 ⓑ 30 ⓑ
31 ⓐ 32 ⓓ 33 ⓒ 34 ⓓ 35 ⓑ
36 ⓒ 37 ⓐ 38 ⓑ 39 ⓐ 40 ⓐ
41 ⓓ 42 ⓑ 43 ⓑ 44 ⓑ 45 ⓒ
46 ⓑ 47 ⓑ 48 ⓑ 49 ⓒ 50 ⓐ
51 ⓑ 52 ⓐ 53 ⓐ 54 ⓐ 55 ⓐ
56 ⓒ 57 ⓒ 58 ⓐ 59 ⓑ 60 ⓒ
61 ⓒ 62 ⓓ 63 ⓑ 64 ⓑ 65 ⓑ
66 ⓑ 67 ⓓ 68 ⓒ 69 ⓓ 70 ⓓ

Answers of Second : Question Bank in Analytic Solid Geometry

First — Questions on 3-dimensional orthogonal coordinate system

1 b	2 d	3 b	4 b	5 d
6 b	7 b	8 c	9 c	10 c
11 a	12 b	13 d	14 b	15 d
16 a	17 c	18 a	19 c	20 c
21 c	22 a	23 b	24 a	25 a
26 d	27 b	28 b	29 b	30 d
31 b	32 d	33 a	34 b	35 b
36 c	37 d	38 b	39 c	40 d
41 d	42 a	43 b	44 d	45 d
46 c	47 a	48 First : b	Second : b	

49 First : b Second : d Third : b

50 First : c Second : b Third : c Fourth : c

Second — Questions on equation of the sphere

1 c	2 b	3 a	4 d	5 d
6 b	7 c	8 b	9 c	10 d
11 c	12 d	13 b	14 b	15 c
16 c	17 b	18 c	19 d	20 b
21 b	22 c	23 d	24 d	25 c
26 b	27 b	28 b	29 d	30 c
31 c	32 c	33 c	34 b	35 d
36 c	37 a	38 d	39 d	40 b
41 d	42 c	43 d	44 a	45 a
46 b	47 b	48 b	49 c	50 d
51 a	52 a	53 b	54 d	55 a
56 c	57 b	58 a	59 d	60 c
61 d	62 c	63 b	64 a	65 c
66 b				

Third — Questions on vector in the space

1 c	2 b	3 c	4 b	5 c
6 b	7 c	8 c	9 b	10 c
11 a	12 c	13 b	14 d	15 d
16 c	17 c	18 b	19 d	20 c
21 c	22 c	23 b	24 b	25 c
26 c	27 a	28 b	29 c	30 a
31 d	32 c	33 b	34 b	35 a
36 b	37 c	38 b	39 b	40 c
41 d	42 b	43 c	44 c	45 a
46 c	47 c	48 a	49 d	50 b
51 a	52 c	53 c	54 a	55 b
56 c	57 b	58 d	59 c	60 b
61 a	62 b	63 c	64 a	65 d
66 c	67 c	68 d	69 d	70 c
71 c	72 b	73 c	74 a	75 d
76 b	77 d	78 d	79 d	80 b
81 b	82 a	83 d	84 b	85 c
86 c	87 b	88 c	89 b	90 b
91 b	92 c	93 c		

Fourth — Questions on scalar product

1 d	2 a	3 a	4 a	5 d
6 c	7 a	8 b	9 c	10 d
11 a	12 d	13 c	14 c	15 c
16 c	17 b	18 c	19 d	20 c
21 c	22 a	23 a	24 d	25 b
26 b	27 d	28 c	29 d	30 c
31 a	32 c	33 d	34 a	35 d
36 d	37 b	38 c	39 c	40 c
41 c	42 a	43 a	44 d	45 b
46 d	47 d	48 a	49 b	50 a
51 d	52 b	53 d	54 c	55 d
56 b	57 a	58 c	59 c	60 b
61 c	62 c	63 b	64 d	65 b
66 c	67 b	68 d	69 c	70 c
71 b	72 a	73 b	74 b	75 b
76 c	77 c	78 c	79 d	80 b
81 a	82 d	83 b	84 a	85 a
86 b	87 b	88 c	89 d	90 d
91 a	92 c	93 b	94 a	95 b
96 a	97 c	98 b	99 b	100 d
101 a	102 First : a	Second : c		103 b
104 b	105 d	106 c	107 c	108 b
109 a	110 a	111 d	112 c	113 a
114 b	115 c	116 a	117 a	

Fifth — Questions on vector product

1 a	2 a	3 d	4 b	5 b
6 c	7 b	8 b	9 d	10 c
11 c	12 c	13 a	14 b	15 b
16 c	17 b	18 b	19 a	20 a
21 c	22 d	23 c	24 d	25 b
26 c	27 d	28 c	29 c	30 c
31 b	32 d	33 c	34 c	35 a
36 b	37 d	38 b	39 d	40 c
41 b	42 d	43 a	44 c	45 c
46 c	47 c	48 a	49 c	50 d
51 c	52 b	53 c	54 a	55 b
56 c	57 d	58 b	59 d	60 b
61 a	62 c	63 c	64 a	65 b
66 c	67 c	68 a	69 d	70 d
71 a	72 b	73 b	74 d	75 c
76 d	77 a	78 c	79 d	80 c
81 b	82 b	83 c	84 a	85 c
86 c				

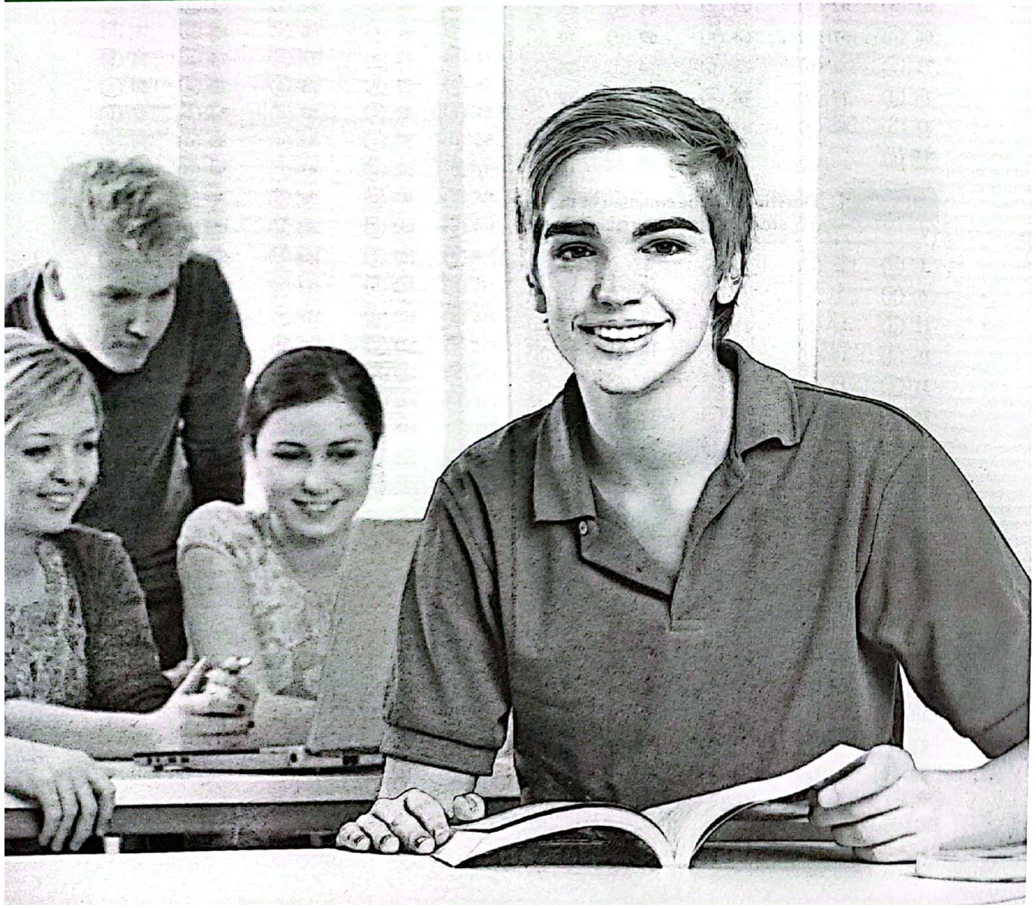
Sixth — Questions on the equation of the straight line in the space

1 a	2 c	3 b	4 a	5 c
6 b	7 a	8 c	9 d	10 b
11 d	12 b	13 c	14 d	15 a
16 d	17 c	18 b	19 a	20 b
21 c	22 d	23 d	24 b	25 c
26 a	27 d	28 c	29 a	30 a
31 d	32 a	33 b	34 d	35 b
36 c	37 c	38 b	39 c	40 d
41 c	42 a	43 a	44 b	45 c
46 c	47 a	48 a	49 c	50 b
51 b	52 b	53 a	54 a	55 d
56 c	57 c	58 d	59 c	60 b
61 a	62 a	63 a	64 a	65 c
66 b	67 d	68 b	69 d	70 b
71 a	72 a	73 a	74 c	75 d
76 a	77 c	78 a	79 c	80 c
81 c	82 d	83 d	84 b	85 c
86 d	87 c	88 b		

Seventh — Questions on equation of the plane in the space

1 d	2 c	3 c	4 b	5 b
6 b	7 c	8 b	9 d	10 a
11 b	12 b	13 c	14 a	15 a
16 d	17 a	18 b	19 c	20 c
21 a	22 b	23 c	24 d	25 d
26 b	27 a	28 c	29 d	30 c
31 c	32 b	33 c	34 d	35 d
36 c	37 c	38 c	39 c	40 d
41 c	42 b	43 b	44 c	45 b
46 b	47 c	48 c	49 a	50 b
51 a	52 c	53 a	54 c	55 d
56 d	57 c	58 c	59 b	60 c
61 d	62 a	63 c	64 d	65 b
66 c	67 c	68 c	69 c	70 a
71 b	72 a	73 b	74 a	75 b
76 a	77 a	78 a	79 a	80 c
81 a	82 c	83 d	84 c	85 b
86 d	87 c	88 d	89 d	90 a
91 b	92 c	93 c	94 b	95 c
96 c	97 c	98 b	99 c	100 c
101 b	102 d	103 d	104 c	105 a
106 d	107 a	108 b	109 c	110 c
111 a	112 b	113 b	114 b	115 d
116 d	117 d	118 a	119 d	120 b
121 c	122 c	123 d	124 c	125 c
126 a	127 b	128 d	129 c	130 d
131 b	132 b	133 d	134 a	135 c
136 c	137 a	138 d	139 c	140 c
141 c	142 d	143 b	144 c	145 b
146 c	147 d	148 a	149 c	150 d
151 b	152 a	153 a	154 c	155 c
156 b	157 a	158 a		

Answers of Practice Exams



Exam 1

1 (d)

Solution :

$z = -2 + 2i$ $\qquad \therefore |z| = \sqrt{4+4} = 2\sqrt{2}$

$\therefore \theta = \pi + \tan^{-1}(-1) = \frac{3}{4}\pi$

$\therefore z = 2\sqrt{2}\, e^{\frac{3\pi}{4} i}$

---

2 (a)

Solution :

$\left(\omega^2 + \frac{1}{\omega}\right)\left(1 + \frac{1}{\omega^2}\right)^2 = (\omega^2 + \omega^2)(1+\omega)^2$

$= 2\omega^2 \times (1 + 2\omega + \omega^2)$

$= 2\omega^2 \times (2\omega - \omega) = 2\omega^2 \times \omega$

$= 2\omega^3 = 2$

---

3 (a)

Solution :

$\left(\sqrt{x} + \frac{1}{x}\right)^8 = \left(x^{\frac{1}{2}} + x^{-1}\right)^8$

$\because 5T_4, T_5, T_7, T_6$ are proportional quantities.

$\therefore \frac{T_5}{5T_4} = \frac{T_6}{T_7}$

$\therefore \frac{1}{5} \times \frac{8-4+1}{4} \times \frac{x^{-1}}{x^{\frac{1}{2}}} = \frac{6}{8-6+1} \times \frac{x^{\frac{1}{2}}}{x^{-1}}$

$\therefore \frac{1}{4} x^{\frac{-3}{2}} = 2x^{\frac{3}{2}}$ $\qquad \therefore x^3 = \frac{1}{8}$

$\therefore x = \frac{1}{2}$

---

4 (d)

Solution :

$\because \cos\theta = \frac{|\overrightarrow{n_1} \cdot \overrightarrow{n_2}|}{\|\overrightarrow{n_1}\| \|\overrightarrow{n_2}\|}$ $\qquad \therefore \frac{1}{2} = \frac{|(1, 1, 0) \cdot (0, k, 1)|}{\sqrt{(1)^2 + (1)^2}\sqrt{k^2 + (1)^2}}$

$\therefore 2|k| = \sqrt{2}\sqrt{k^2+1}$

$\therefore 4k^2 = 2(k^2+1)$ $\qquad \therefore 2k^2 = k^2 + 1$ $\qquad \therefore k^2 = 1$

$\therefore k = 1$ (because $k > 0$)

---

5 (b)

Solution :

$\cos 2\theta_x + \cos 2\theta_y + \cos 2\theta_z$

$= 2\cos^2\theta_x - 1 + 2\cos^2\theta_y - 1 + 2\cos^2\theta_z - 1$

$= 2(\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z) - 3$

$= 2 \times 1 - 3 = -1$

---

6 (a)

Solution :

Let $z = x + yi$ $\qquad \therefore \bar{z} = x - yi$

$\therefore z + \bar{z} = 2x$ $\qquad \therefore 2x = 2e^{\pi i}$

$\therefore x = -1$

$\therefore$ z could be any complex number has real part $= -1$

$\therefore$ z could be $e^{\pi i}$

---

7 (c)

Solution :

$\because -xyz = -100$ $\qquad \therefore xyz = 100$ $\qquad (1)$

$, \frac{1}{2} xy \sin z = 6.25$ $\qquad (2)$

Divide (1) by (2) : $\therefore \frac{z}{\frac{1}{2}\sin z} = 16$ $\qquad \therefore \frac{z}{\sin z} = 8$

$\therefore 2r = 8$ cm.

---

8 (a)

Solution :

Find the equation of the straight line $\overleftrightarrow{BC}$ then the direction vector of the straight line is $\overrightarrow{BC}$

$= (7, -2, 5) - (1, 2, 3) = (6, -4, 2)$

$\therefore$ Equation of the straight line $\overleftrightarrow{BC}$ is :

$\vec{r} = (1, 2, 3) + t(6, -4, 2)$

$\therefore$ Let the point of projection is :

$D = (1 + 6t, 2 - 4t, 3 + 2t)$

$\therefore \overrightarrow{AD} = (1 + 6t, 2 - 4t, 3 + 2t) - (0, 9, 6)$

$= (1 + 6t, -7 - 4t, -3 + 2t)$

$\because \overrightarrow{AD} \perp \overrightarrow{BC}$

$\therefore \overrightarrow{AD} \cdot \overrightarrow{BC} = 0$

$\therefore (1+6t, -7-4t, -3+2t) \cdot (6, -4, 2) = 0$

$\therefore 6 + 36t + 28 + 16t + (-6) + 4t = 0$

$\therefore 56t = -28 \qquad \therefore t = -\frac{1}{2}$

$\therefore$ The point $D = (-2, 4, 2)$

9 (a)

Solution :

$$A^2 = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

$$= \begin{pmatrix} \cos^2\theta - \sin^2\theta & \cos\theta\sin\theta + \sin\theta\cos\theta \\ -\sin\theta\cos\theta - \sin\theta\cos\theta & -\sin^2\theta + \cos^2\theta \end{pmatrix}$$

$$= \begin{pmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{pmatrix}$$

$$\therefore |A^2| = \begin{vmatrix} \cos 2\theta & \sin 2\theta \\ -\sin 2\theta & \cos 2\theta \end{vmatrix} = \cos^2 2\theta + \sin^2 2\theta = 1$$

$$\therefore (A^2)^{-1} = \frac{1}{1}\begin{pmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix} = \begin{pmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix}$$

10 (c)

Solution :

$$\because \begin{vmatrix} 1 & 1 & 1 \\ 4 & -k & -k \\ 3 & 2 & -4 \end{vmatrix} = 1(4k+2k) - 1(-16+3k) + 1(8+3k)$$

$$= 6k + 16 - 3k + 8 + 3k$$

$$= 6k + 24$$

the system of equations has unique solution should be $6k + 24 \neq 0$

$\therefore k \neq -4 \qquad \therefore k \in \mathbb{R} - \{-4\}$

11 (b)

Solution :

$\because a_{xy} = 2x - y \qquad \therefore (A_{xy}) = \begin{pmatrix} 1 & 0 & -1 \\ 3 & 2 & 1 \\ 5 & 4 & 3 \end{pmatrix}$

$, \because |A_{xy}| = \begin{vmatrix} 1 & 0 & -1 \\ 3 & 2 & 1 \\ 5 & 4 & 3 \end{vmatrix} = \text{zero}$

$, \because \begin{vmatrix} 1 & 0 \\ 3 & 2 \end{vmatrix} = 2 \neq \text{zero}$

$\therefore RK(A_{xy}) = 2$

12 (b)

Solution :

$\overrightarrow{A} + \overrightarrow{B} = (0, 2, 0) \quad , \quad \overrightarrow{B} + \overrightarrow{C} = (-2, 3, 0)$

$$(\overrightarrow{A} + \overrightarrow{B}) \times (\overrightarrow{B} \times \overrightarrow{C}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 2 & 0 \\ -2 & 3 & 0 \end{vmatrix} = 4\hat{k}$$

$\therefore$ the normal direction vector $= \pm\frac{4\hat{k}}{4} = \pm\hat{k}$

13 (a)

Solution :

$\because \overline{n}_1 = (2, c, 4) , \overline{n}_2 = (a+2, 6, b-2)$

$, \because$ the two planes are parallel

$\therefore \frac{2}{a+2} = \frac{c}{6} = \frac{4}{b-2} \qquad \therefore 2(b-2) = 4(a+2)$

$\therefore 2b - 4 = 4a + 8 \qquad \therefore 4a - 2b = -12$

$\therefore 2a - b = -6$

14 (b)

Solution :

direction angles of the straight line is $(90°, 135°, 45°)$

$\therefore$ direction vector of the straight line $\overrightarrow{d}$

$= (\cos 90°, \cos 135°, \cos 45°)$

$= \left(0, \frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

$\therefore$ the vector $(0, -1, 1)$ is direction vector also.

$\therefore$ the vector form of the equation of the straight line is $\overline{r} = (2, -1, 4) + t(0, -1, 1)$

15 (a)

Solution :

$\because 2\,{}^{n+1}C_r = {}^{n+1}P_r \qquad \therefore 2 \times \frac{{}^{n+1}P_r}{\lfloor r} = {}^{n+1}P_r$

$\therefore \lfloor r = 2 \qquad \therefore r = 2$

$\because \frac{{}^{n}C_{r+1}}{{}^{n}C_r} = \frac{5}{3} \qquad \therefore \frac{n-(r+1)+1}{r+1} = \frac{5}{3}$

$\therefore \frac{n-2}{3} = \frac{5}{3} \qquad \therefore n = 7$

$\therefore {}^{n}C_r + {}^{n}P_r = {}^{7}C_2 + {}^{7}P_2 = 63$

16 (a)

Solution :

$\because \left(x^2 + 2 + \frac{1}{x^2}\right)^6 = \left(\left(x + \frac{1}{x}\right)^2\right)^6 = \left(x + \frac{1}{x}\right)^{12}$

$, \because T_{r+1} = {}^{12}C_r \left(\frac{1}{x}\right)^r (x)^{12-r} = {}^{12}C_r\, x^{12-2r}$

$,$ put $12 - 2r = 2 \qquad \therefore r = 5$

$\therefore T_6$ is the term which contains x^2

$\therefore$ The coefficient of $T_6 = {}^{12}C_5$

17 (b)

Solution :

$\because \overrightarrow{A} + \overrightarrow{BC} = (4, 12, 9)$

$\therefore \overrightarrow{A} + \overrightarrow{C} - \overrightarrow{B} = (4, 12, 9)$

$\therefore \overrightarrow{C} = (4, 12, 9) - (0, -1, 3) + (4, -2, 1)$

$= (8, 11, 7)$

$\therefore \overrightarrow{C} = 8\hat{i} + 11\hat{j} + 7\hat{k}$

18 (c)

Solution :

$BO = CE = 4 \sin 60° = 4 \times \frac{\sqrt{3}}{2} = 2\sqrt{3}$ cm.

$\therefore MB = MC = \frac{4}{3}\sqrt{3}$ cm.

$, m(\angle BMC) = 120°$

$\therefore \overrightarrow{MB} \cdot \overrightarrow{CM} = \overrightarrow{MB} \cdot (-\overrightarrow{MC})$

$= -\|\overrightarrow{MB}\|\|\overrightarrow{MC}\| \cos 120°$

$= \frac{-4}{3}\sqrt{3} \times \frac{4}{3}\sqrt{3} \times \frac{-1}{2} = \frac{8}{3}$

19 (c)

Solution :

${}^{n}C_2 = \frac{1}{2} n(n-1)$

$= \frac{1}{2}\,{}^{m}C_2 ({}^{m}C_2 - 1)$

$= \frac{1}{2} \times \frac{1}{2} m(m-1)\left(\frac{1}{2} m(m-1) - 1\right)$

$= \frac{1}{8} m(m-1)(m-2)(m+1)$

$= 3 \times \frac{(m+1)(m)(m-1)(m-2)}{\lfloor 4} = 3\,{}^{m+1}C_4$

20 (b)

Solution :

$\because \overline{n} \,//$ cartesian plane yz

$\therefore (a, 4, c) \perp (1, 0, 0)$

$\therefore (a, 4, c) \cdot (1, 0, 0) =$ zero

$\therefore a =$ zero $\qquad \therefore a =$ zero

$\because \|\overline{n}\| = 5 \qquad \therefore \sqrt{0^2 + 4^2 + c^2} = 5$

$\therefore c^2 = 9$

21 (c)

Solution :

$$\because \begin{vmatrix} a^2 & c^2 & -b^2 \\ 1 & 1 & -2 \\ 0 & 1 & -1 \end{vmatrix} = ac$$

By using elements of c_1 : the expansion is :

$a^2(-1+2) - 1(-c^2 + b^2) = ac$

$\therefore a^2 + c^2 - b^2 = ac$

$\therefore \frac{a^2 + c^2 - b^2}{2ac} = \frac{1}{2} \qquad \therefore \cos B = \frac{1}{2}$

$\therefore m(\angle B) = 60°$

22 (c)

Solution :

$z = k\left(\sin\frac{4\pi}{3} - i\cos\frac{4\pi}{3}\right)$

$= k\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right)$

$\therefore z^6 = k^6\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right)^6$

$= k^6(\cos 5\pi + i\sin 5\pi) = -k^6$

23 (a)

Solution :

$\because AM = \sqrt{(3-1)^2 + (5-2)^2 + (1+5)^2} = 7$ length unit

$\therefore r = 7 - 2 = 5$ length unit

24 (a)

Solution :

$\left(1 - \frac{1}{x}\right)^n (1-x)^n = \frac{(x-1)^n}{x^n} \times (-1)^n \times (x-1)^n$

$= \frac{(-1)^n}{x^n} \times (x-1)^{2n}$

$\therefore$ order of the middle term $= \frac{2n+2}{2} = n+1$

$\therefore T_{n+1} = \frac{(-1)^n}{\cancel{x^n}} \times {}^{2n}C_n \times (-1)^n \times \cancel{x^n}$

$= (-1)^{2n} \times {}^{2n}C_n = {}^{2n}C_n$

25 (c)

Solution :

number of numbers in case zero in unit digit

$= 1 \times 3 \times 2 = 6$ numbers

number of members in case 2 in unit digit.

$= 1 \times 2 \times 2 = 4$ number.

$\therefore$ number of numbers $= 6 + 4 = 10$ numbers.

Exam 2

1 (d)

Solution :

$$\frac{\text{projection of } \vec{A} \text{ in direction } \vec{B}}{\text{projection of } \vec{B} \text{ in direction } \vec{A}} = \frac{\vec{A}.\vec{B}}{\|\vec{B}\|} \div \frac{\vec{A}.\vec{B}}{\|\vec{A}\|} = \frac{\|\vec{A}\|}{\|\vec{B}\|} = \frac{\sqrt{4+9+36}}{\sqrt{4+4+1}} = \frac{7}{3}$$

2 (d)

Solution :

$\because$ The majority of the committee are women and the committee contains both genders

$\therefore$ The committee formed from (3 women and 2 men or 4 women and a man)

$= {}^6C_3 \times {}^9C_2 + {}^6C_4 \times {}^9C_1 = 855$

3 (b)

Solution :

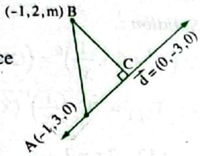
$$\begin{vmatrix} 1 & \omega & \omega-1 \\ 1 & -1 & \omega+1 \\ 1 & \omega & \omega \end{vmatrix} \left(\text{by doing } (R_2 - R_1), (R_3 - R_1)\right)$$

$$= \begin{vmatrix} 1 & \omega & \omega-1 \\ 0 & -1-\omega & 2 \\ 0 & 0 & 1 \end{vmatrix}$$

$= 1 \times -(1+\omega) \times 1 = -(-\omega^2) = \omega^2$

4 (c)

Solution :



$\because$ The perpendicular distance

$(BC) = \frac{\|\overrightarrow{AB} \times \vec{d}\|}{\|\vec{d}\|}$

$\therefore \|\overrightarrow{AB} \times \vec{d}\| = 8 \times 3 = 24$

$, \because \overrightarrow{AB} = \vec{B} - \vec{A} = (0, -1, m)$

$$\therefore \overrightarrow{AB} \times \vec{d} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & -1 & m \\ 0 & -3 & 0 \end{vmatrix} = 3m\hat{i}$$

$\therefore |3m| = 24 \qquad \therefore 3m = 24$

$\therefore m = 8$ where $m \in \mathbb{R}^+$

5 (a)

Solution :

$\because {}^{n+1}P_r > {}^{n+1}P_{r-1}$

$\therefore \frac{(n+1)!}{(n+1-r)!} > \frac{(n+1)!}{(n+1-r+1)!}$

$\therefore (n-r+1)! < (n-r+2)!$

$\therefore (n-r+1)! < (n-r+2)\,(n-r+1)!$

$\therefore n - r + 2 > 1 \qquad \therefore n > r - 1$

6 (d)

Solution :

${}^5C_0 + {}^5C_1 x + {}^5C_2 x^2 + {}^5C_3 x^3 + \dots + {}^5C_5 x^5 = 1024$

$\therefore (1+x)^5 = 1024 \qquad \therefore 1 + x = 4$

$\therefore x = 3$

7 (d)

Solution :

$$\overrightarrow{AB} \times \overrightarrow{AD} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 2 & -1 \\ -1 & 2 & -3 \end{vmatrix} = -4\hat{i} + 7\hat{j} + 6\hat{k}$$

$\therefore$ area of the parallelogram $= \|\overrightarrow{AB} \times \overrightarrow{AD}\|$

$= \sqrt{16 + 49 + 36}$

$= \sqrt{101}$ square unit.

8 (a)

Solution :

$\left(\frac{a}{\omega} - \frac{a}{\omega^2} + \frac{3a}{\omega^4} - \frac{3a}{\omega^5}\right)^2$

$= \left(\frac{a}{\omega} - \frac{a}{\omega^2} + \frac{3a}{\omega} - \frac{3a}{\omega^2}\right)^2$

$= \left(\frac{4a}{\omega} - \frac{4a}{\omega^2}\right)^2 = 16a^2\left(\frac{1}{\omega} - \frac{1}{\omega^2}\right)^2$

$= 16a^2(\omega^2 - \omega)^2 = 16a^2 \times \left(\pm\sqrt{3}\,i\right)^2$

$= 16a^2 \times 3i^2 = -48a^2$

9 (b)

Solution :

$\because \overleftrightarrow{AB}$ is a tangent to the circle

$\therefore (AB)^2 = BC \times BD = BC \times (BC + CD)$

$= (BC)^2 + BC \times CD$

$, \because$ the determinant $= (AB)^2 + (BC)^2 + CD \times BC$

$= (AB)^2 + (AB)^2 = 2(AB)^2$

$\therefore 2(AB)^2 = 32 \qquad \therefore (AB)^2 = 16$

$\therefore AB = 4$ length units

10 (a)

Solution :

$\frac{\text{coefficient of } T_{11}}{\text{coefficient of } T_{10}} \geq 1 \qquad \therefore \frac{20-10+1}{10} \times \frac{1}{a} \geq 1$

$\therefore \frac{11}{10} \times \frac{1}{a} \geq 1 \qquad \therefore \frac{1}{a} \geq \frac{10}{11}$

$\therefore a \leq \frac{11}{10}$

$, \frac{\text{coefficient of } T_{12}}{\text{coefficient of } T_{11}} \leq 1 \qquad \therefore \frac{20-11+1}{11} \times \frac{1}{a} \leq 1$

$\therefore \frac{10}{11} \times \frac{1}{a} \leq 1 \qquad \therefore \frac{1}{a} \leq \frac{11}{10}$

$\therefore a \geq \frac{10}{11}$

$\therefore \frac{10}{11} \leq a \leq \frac{11}{10} \qquad \therefore a \in \left[\frac{10}{11}, \frac{11}{10}\right]$

11 (b)

Solution :

$M_1 = (-3, 4, 1)$, $M_2 = (5, -2, 1)$

$\therefore$ Midpoint of $\overline{M_1M_2} = \left(\frac{5-3}{2}, \frac{4-2}{2}, \frac{1+1}{2}\right)$

$= (1, 1, 1)$

$\therefore 2a - 3a + 4a + 6 = 0$

$\therefore 3a = -6 \qquad \therefore a = -2$

12 (d)

Solution :

$\because \vec{A} \times \vec{B} = \pm\|\vec{A} \times \vec{B}\|\vec{N}$

$= \pm 5\left(\frac{3}{5}, 0, \frac{4}{5}\right) = \pm(3, 0, 4)$

$\therefore (3\vec{A} + \vec{B}) \times (4\vec{A} + 2\vec{B}) = \vec{O} + 6\vec{A} \times \vec{B}$

$+ 4\vec{B} \times \vec{A} + \vec{O}$

$= 2\vec{A} \times \vec{B} = \pm 2(3, 0, 4) = \pm(6, 0, 8)$

13 (d)

Solution :

$\because T_{r+1} = {}^7C_r \left(\frac{a}{x^2}\right)^r (x)^{7-r}$

$= {}^7C_r\, a^r x^{7-3r}$

at $7 - 3r = 4 \qquad \therefore r = 1$

$\therefore$ coefficient of $x^4 = {}^7C_1\, a^1 = 49 \qquad \therefore a = 7$

14 (d)

Solution :

$\because \overrightarrow{DC} = \vec{C} - \vec{D} = (0, -4, 6)$

$, \because \vec{A} \,//\, \overrightarrow{DC}$

$\therefore \vec{A} = \sqrt{13}$ (the unit vector in $\overrightarrow{DC}$ direction)

$= \sqrt{13} \times \frac{(0, -4, 6)}{\sqrt{(-4)^2 + (6)^2}} = (0, -2, 3)$

$\therefore \vec{A} \times \vec{B} = (0, -2, 3) \times (-2, 3, 5)$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & -2 & 3 \\ -2 & 3 & 5 \end{vmatrix} = -19\hat{i} - 6\hat{j} - 4\hat{k}$$

15 (d)

Solution :

$\therefore 3z_1 \times z_2 = 3(\cos\theta + i\sin\theta)(\cos 2\theta + i\sin 2\theta)$

$= 3(\cos 3\theta + i\sin 3\theta)$

$\therefore$ The principle amplitude of the complex number $(3z_1 \times z_2)$ is 3θ

16 (c)

Solution :

$\because {}^{n}C_r : {}^{n-1}C_r = 3 : 1$

$\therefore \frac{\lfloor n}{\lfloor r \lfloor n-r} \times \frac{\lfloor n-1-r \lfloor r}{\lfloor n-1} = \frac{3}{1}$

$\therefore \frac{n \lfloor n-1}{(n-r)\lfloor n-r-1 \lfloor r} \times \frac{\lfloor n-1-r \lfloor r}{\lfloor n-1} = \frac{3}{1}$

$\therefore \frac{n}{n-r} = \frac{3}{1}$ $\quad \therefore 3n - 3r = n$

$\therefore 2n = 3r$ $\quad \therefore \frac{n}{r} = \frac{3}{2}$

$\therefore \lfloor 4\frac{n}{r} = \lfloor 6 = 720$

17 (a)

Solution :

the cubic roots of unity represented on Argand diagram by the points $(1, 0), \left(\frac{-1}{2}, \frac{\sqrt{3}}{2}\right)$, $\left(\frac{-1}{2}, \frac{-\sqrt{3}}{2}\right)$ and they are vertices of an equilateral triangle with side length

$= \sqrt{\left(1+\frac{1}{2}\right)^2 + \left(0 - \frac{\sqrt{3}}{2}\right)^2} = \sqrt{3}$ length unit.

$\therefore$ the area of the triangle $= \frac{\sqrt{3}}{4} \times \left(\sqrt{3}\right)^2$

$= \frac{3\sqrt{3}}{4}$ square unit.

18 (c)

Solution :

The equation of the plane is $\begin{vmatrix} x-2 & y-3 & z-5 \\ -1-2 & 3-3 & 1-5 \\ 4-2 & 3-3 & -2-5 \end{vmatrix} = 0$

$\therefore \begin{vmatrix} x-2 & y-3 & z-5 \\ -3 & 0 & -4 \\ 2 & 0 & -7 \end{vmatrix} = 0$

$\therefore -(y-3)\begin{vmatrix} -3 & -4 \\ 2 & -7 \end{vmatrix} = 0$

$\therefore y - 3 = 0$ $\quad \therefore y = 3$

19 (c)

Solution :

$\overrightarrow{n_1} = (0, 0, 1), \overrightarrow{n_2} = (1, 0, \sqrt{3})$

$\cos\theta = \frac{|(0,0,1).(1,0,\sqrt{3})|}{\sqrt{0+0+1}\sqrt{1+0+3}} = \frac{\sqrt{3}}{2}$

$\therefore \theta = 30°$

20 (a)

Solution :

$z_1 = 3(\cos 300° + i \sin 300°)$

$= 3\left(\cos(-60°) + i \sin(-60°)\right)$

$, z_2 = 2(\sin 240° + i \cos 240°)$

$= 2\left(\cos(90° - 240°) + i \sin(90° - 240°)\right)$

$= 2\left(\cos(-150) + i \sin(-150°)\right)$

$z_1 z_2 = 3 \times 2\left(\cos(-60° - 150°) + i \sin(-60° - 150°)\right)$

$= 6\left(\cos(-210) + i \sin(-210)\right)$

$= 6\left(\cos(150) + i \sin(150)\right)$

$\therefore z_1 z_2 = 6\, e^{\frac{5}{6}\pi i}$

21 (a)

Solution :

$\begin{vmatrix} x & y & y \\ y & x & y \\ y & y & x \end{vmatrix}$ (by doing $C_1 + C_2 + C_3$)

$= \begin{vmatrix} x+2y & y & y \\ x+2y & x & y \\ x+2y & y & x \end{vmatrix}$

$= (x+2y)\begin{vmatrix} 1 & y & y \\ 1 & x & y \\ 1 & y & x \end{vmatrix}$

(by doing $R_2 - R_1, R_3 - R_1$)

$= (x+2y)\begin{vmatrix} 1 & y & y \\ 0 & x-y & 0 \\ 0 & 0 & x-y \end{vmatrix}$

22 (a)

Solution :

$A^* = \begin{pmatrix} 3 & 2 & -1 : 4 \\ 1 & 1 & -1 : 3 \\ 1 & 0 & -2 : 0 \end{pmatrix}$

$\because \begin{vmatrix} 3 & 2 & -1 \\ 1 & 1 & -1 \\ 1 & 0 & -2 \end{vmatrix} = 3(-2-0) - 2(-2+1) - 1(0-1)$

$= -3 \neq 0$

$\therefore RK(A^*) = 3$ $\quad \therefore 2 < RK(A^*) < 4$

23 (b)

Solution :

$\because$ the straight line passes through $(4, 4, 7)$

$\therefore \frac{2 \times 4 - \ell}{4} = \frac{4-2}{2} = \frac{7-2}{5}$

$\therefore \frac{8-\ell}{4} = 1$ $\quad \therefore \ell = 4$

$\therefore$ the equation of the straight line is :

$\frac{2x-4}{4} = \frac{y-2}{2} = \frac{z-2}{5}$

i.e. $\frac{x-2}{2} = \frac{y-2}{2} = \frac{z-2}{5}$

$\therefore \vec{d} = (2, 2, 5)$

$\therefore$ the vector $(m, n, 10)$ parallels $(2, 2, 5)$

$\therefore \frac{m}{2} = \frac{n}{2} = \frac{10}{5}$ $\quad \therefore m = n = 4$

$\therefore \ell + m + n = 4 + 4 + 4 = 12$

24 (b)

Solution :

the sphere passes through the point $(6, 3, -3)$ and touches the cartesian planes xy, xz, yz

$\therefore$ let its centre coordinates is $(r, r, -r)$

$\because$ its equation is :

$(x-r)^2 + (y-r)^2 + (z+r)^2 = r^2$

$\therefore (6-r)^2 + (3-r)^2 + (-3+r)^2 = r^2$

$\therefore 36 - 12r + r^2 + 9 - 6r + r^2 + 9 - 6r + r^2 = r^2$

$\therefore 2r^2 - 24r + 54 = 0$

$\therefore r^2 - 12r + 27 = 0$ $\quad \therefore r = 3$ length unit

or $r = 9$ length unit.

25 (a)

Solution :

$A^{-1} = \frac{1}{|A|}(\text{Adj}(A))$ $\quad \therefore \text{Adj}(A) = |A|\, A^{-1}$

Exam 3

1 (a)

Solution :

$T_{r+1} = {}^{12}C_r \left(\frac{2}{x^3}\right)^r \times (x^2)^{12-r} = {}^{12}C_r \times 2^r \times x^{24-5r}$

Put : $24 - 5r = -1$ $\quad \therefore 5r = 25$ $\quad \therefore r = 5$

$\therefore T_6 = {}^{12}C_5 \times 2^5 \times \frac{1}{x}$ $\quad \therefore a = {}^{12}C_5 \times 2^5$

2 (c)

Solution :

$\because \overrightarrow{n_1} = (3, -6, 6), \overrightarrow{n_2} = (1, 0, 1)$

$\therefore \cos\theta = \frac{|(3,-6,6).(1,0,1)|}{\sqrt{9+36+36}\sqrt{1+1}} = \frac{1}{\sqrt{2}}$

$\therefore \theta = 45°$

3 (b)

Solution :

$|2\, \text{Adj}(A)| = |2|A|A^{-1}|$

$= |2 \times 5 \times A^{-1}|$

$= 10^3 |A^{-1}|$

$= 10^3 \times \frac{1}{|A|} = 10^3 \times \frac{1}{5}$

$= 200$

4 (a)

Solution :

the straight line : $2x - 4 = \frac{2y-8}{3} = \frac{2z-14}{5}$

(divided by 2)

$\therefore \ell : \frac{x-2}{1} = \frac{y-4}{3} = \frac{z-7}{5}$

$\therefore \vec{d} = (1, 3, 5)$ and passes through $(2, 4, 7)$

i.e. the perpendicular distance = zero.

5 (b)

Solution :

$x = \frac{4}{\sqrt{3}+i} \times \frac{\sqrt{3}-i}{\sqrt{3}-i} = \frac{4(\sqrt{3}-i)}{3+1} = \sqrt{3} - i$

$, y = \frac{2(\cos 0° + i \sin 0°)}{\cos\left(\frac{-\pi}{6}\right) + i \sin\left(\frac{-\pi}{6}\right)} = 2\left(\cos\frac{\pi}{6} + i \sin\frac{\pi}{6}\right)$

$= 2\left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right) = \sqrt{3} + i$

$\therefore x, y$ are two conjugate

6 (a)

Solution :

$\vec{A} = (-2k, 2k, k)$

$\hat{u}_A = \frac{(-2k, 2k, k)}{\sqrt{4k^2 + 4k^2 + k^2}} = \left(\frac{-2}{3}, \frac{2}{3}, \frac{1}{3}\right)$

$\therefore$ direction cosines of $\vec{A} = \left(\frac{-2}{3}, \frac{2}{3}, \frac{1}{3}\right)$

7 (a)

Solution :

$\overrightarrow{n_1}=(3,-1,1)$, $\overrightarrow{n_2}=(1,4,-2)$

$$\therefore \overrightarrow{n_1}\times\overrightarrow{n_2}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & -1 & 1\\ 1 & 4 & -2\end{vmatrix}=-2\hat{i}+7\hat{j}+13\hat{k}$$

, then the line of intersection of the two planes is parallel to the vector $(-2,7,13)$

---

8 (c)

---

9 (a)

Solution :

$x+\frac{1}{x}=i$ (multiply by x)

$\therefore x^2-ix+1=0$

$\therefore$ product of the two roots $(z_1 z_2)=1$

---

10 (b)

Solution :

$\frac{x+3}{3}=\frac{y-2}{-2}=\frac{z+1}{1}=k$

$\therefore x=3k-3$, $y=-2k+2$, $z=k-1$

$\therefore 4(3k-3)+5(-2k+2)+3(k-1)-5=0$

$12k-12-10k+10+3k-3-5=0$

$\therefore k=2$

$\therefore$ intersection point of the straight line and the plane is $(3,-2,1)$

---

11 (b)

Solution :

$$\therefore 4k\,\underline{|2k-1}=\frac{32}{9}\times\frac{{}^{11}C_3+{}^{11}C_4}{{}^{12}C_3}+40$$

$$\therefore 2\times2k\,\underline{|2k-1}=\frac{32}{9}\times\frac{{}^{12}C_4}{{}^{12}C_3}+40$$

$\therefore 2\,\underline{|2k}=\frac{32}{9}\times\frac{9}{4}+40=48$ $\therefore \underline{|2k}=24=\underline{|4}$

$\therefore 2k=4$ $\therefore k=2$

---

12 (a)

Solution :

the complex Number is $\sqrt{k}-i\sqrt{k}$

$\therefore r=\sqrt{(\sqrt{k})^2+(-\sqrt{k})^2}=\sqrt{2k}$

A lies in the fourth quadrant

$\therefore \tan\theta=\frac{-\sqrt{k}}{\sqrt{k}}=-1$

$\therefore \theta=\frac{-\pi}{4}$

$\therefore$ the number is $\sqrt{2k}\,e^{\frac{-\pi}{4}i}$

---

13 (a)

Solution :

$\overrightarrow{CA}=\overrightarrow{CB}+\overrightarrow{BA}=(-\overrightarrow{j}+\overrightarrow{i}-\overrightarrow{k})+(-2\overrightarrow{i}+3\overrightarrow{j})$

$=-\overrightarrow{i}+2\overrightarrow{j}-\overrightarrow{k}$

---

14 (d)

Solution :

$\frac{\underline{|7}}{\underline{|7-r}}=\frac{\underline{|5}}{\underline{|5-r}}$ $\therefore \frac{7\times6\,\cancel{\underline{|5}}}{(7-r)(6-r)\,\cancel{\underline{|5-r}}}=\frac{\cancel{\underline{|5}}}{\cancel{\underline{|5-r}}}$

$\therefore (7-r)(6-r)=7\times6$

$\therefore {}^{7-r}P_2={}^{7}P_2\Rightarrow 7-r=7$

$\therefore r=0$

---

15 (a)

Solution :

the straight lines which passes through a point from the first set $\{B,C,D,E\}$ and a point from the second set $\{X,Y,Z,L,M\}$ without repeating adding to the two straight lines which carrying the two rays :

its number $=5\times4+2=22$ lines

---

16 (c)

Solution :

$\hat{i}\times(\hat{i}+\hat{j})=\hat{i}\times\hat{i}+\hat{i}\times\hat{j}=0+\hat{k}=\hat{k}$

$\therefore$ the area of parallelogram $=\|\hat{i}\times(\hat{i}+\hat{j})\|$

$=\|\hat{k}\|=1$ square unit.

---

17 (c)

Solution :

$\because RK(A)=3$ $\therefore |A|\neq0$

$$\therefore \begin{vmatrix}1 & 2 & 3\\ 2 & m & 6\\ 5 & 7 & 9\end{vmatrix}\neq \text{zero}$$

$\therefore -2(18-30)+m(9-15)+7(0)\neq0$

$\therefore m\neq4$ $\therefore m\in\mathbb{R}-\{4\}$

18 (a)

Solution :

$\because 3\overrightarrow{A}\,.\,2\overrightarrow{B}=12$ $\therefore 6(\overrightarrow{A}\,.\,\overrightarrow{B})=12$

$\therefore \overrightarrow{A}\,.\,\overrightarrow{B}=2$

$\therefore -\overrightarrow{B}\,.\,5\overrightarrow{A}=-5(\overrightarrow{B}\,.\,\overrightarrow{A})=-5\times2=-10$

---

19 (c)

Solution :

$M=\left(\frac{1-2+1}{3},\frac{2+0+4}{3},\frac{4+5+0}{3}\right)=(0,2,3)$

$\therefore \overrightarrow{AM}$ is the direction vector of the straight line

$\overrightarrow{AM}=\overrightarrow{M}-\overrightarrow{A}=(-1,0,-1)$

$\therefore \overrightarrow{r}=(1,2,4)+t(-1,0,-1)$

---

20 (b)

Solution :

$\because$ coefficient of T_{r+2} = coefficient of T_{r+4}

$\therefore {}^{20}C_{r+1}={}^{20}C_{r+3}$

$\therefore r+1=r+3$ $\therefore 1=3$ (refused)

or $r+1+r+3=20$ $\therefore r=8$

---

21 (b)

Solution :

$T_{r+1}={}^{7n}C_r\left(\frac{-k}{x^2}\right)^r(x^5)^{7n-r}={}^{7n}C_r(-k)^r x^{35n-7r}$

Put $35n-7r=0$ $\therefore r=5n$

$\therefore$ The term free of x is T_{5n+1}

---

22 (a)

Solution :

$$\because A=\begin{pmatrix}3 & 1 & -1\\ 5 & 2 & -3\\ -1 & -3 & 9\end{pmatrix}$$

$$\because |A|=\begin{vmatrix}3 & 1 & -1\\ 5 & 2 & -3\\ -1 & -3 & 9\end{vmatrix}$$

$=3(18-9)-1(45-3)-1(-15+2)$

$=-2\neq0$

$\therefore RK(A)=3$

$$\because A^*=\begin{pmatrix}3 & 1 & -1 & :0\\ 5 & 2 & -3 & :2\\ -1 & -3 & 9 & :5\end{pmatrix}$$

in the order 3×4 and is non-zero matrix.

$\because |A|\neq0$ $\therefore RK(A^*)=3$

$\therefore RK(A)=RK(A^*)=3=$ Number of unknown.

$\therefore$ the system of the equations has a unique solution

---

23 (c)

Solution :

$$\because 2\times2\begin{vmatrix}a & b & c\\ 1 & -2 & 3\\ e & f & d\end{vmatrix}=2\times2\times8$$

$$\therefore \begin{vmatrix}2a & 2b & 2c\\ 2 & -4 & 6\\ e & f & d\end{vmatrix}=32 \qquad \therefore \begin{vmatrix}2 & -4 & 6\\ 2a & 2b & 2c\\ e & f & d\end{vmatrix}=-32$$

---

24 (b)

Solution :

$$\begin{vmatrix}a+2 & 3 & \sin C\\ 1 & b & 0\\ 2 & 3 & \sin C\end{vmatrix}=12$$

(by doing R_1-R_3)

$$\therefore \begin{vmatrix}a & 0 & 0\\ 1 & b & 0\\ 2 & 3 & \sin C\end{vmatrix}=12$$

$\therefore ab\sin C=12$

$\therefore$ area of the triangle ABC

$=\frac{1}{2}ab\sin C=6$ square unit

---

25 (d)

Solution :

$\because M_1M_2=\sqrt{(-3+2)^2+(2-1)^2+(-6\sqrt{2}+5\sqrt{2})^2}$

$=2$ length units

, $\because r_1-r_2=M_1M_2$ $\therefore 8-r_2=2$

$\therefore r_2=6$ length units.

Exam 4

1 (a)

Solution :

Number of ways $={}^{20}C_1+{}^{20}C_2+{}^{20}C_3+\dots+{}^{20}C_{20}$

$=({}^{20}C_0+{}^{20}C_1+{}^{20}C_2+\dots+{}^{20}C_{20})-{}^{20}C_0$

$=2^{20}-1$

2 (c)

Solution :

$$\frac{z_1}{z_2}=\frac{15\left(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}\right)}{3(\cos\theta+i\sin\theta)}=5\left(\cos\left(\frac{\pi}{2}-\theta\right)+i\sin\left(\frac{\pi}{2}-\theta\right)\right)$$

3 (d)

Solution :

$r=\sqrt{(3-0)^2+(-1-0)^2+(2-0)^2}=\sqrt{14}$ length unit

∴ The equation of the sphere is : $x^2+y^2+z^2=14$

4 (a)

Solution :

$$\therefore x-y=\frac{1}{1+\omega i}-\frac{\omega+i}{1+\omega^2 i}=\frac{(1+\omega^2 i)-(\omega+i)(1+\omega i)}{(1+\omega i)(1+\omega^2 i)}=\frac{1+\omega^2 i-(\omega+i+\omega^2 i-\omega)}{1+\omega i+\omega^2 i-1}=\frac{1-i}{(\omega+\omega^2)i}=\frac{1-i}{-i}=i+1$$

5 (c)

Solution :

vector $(2,3,4)$ is direction vector of the straight line

∵ the vector $(1,\ell,m)$ is normal vector of the plane.

$\therefore \frac{2}{1}=\frac{3}{\ell}=\frac{4}{m}$ $\quad\therefore \ell=\frac{3}{2}$, $m=2$

$\therefore \ell\times m=3$

6 (d)

Solution :

$$z_1+z_2=e^{5+k\pi i}+e^{(5+ki)\pi}=e^{5+k\pi i}+e^{5\pi+\pi k i}=\left(e^5+e^{5\pi}\right)e^{(k\pi)i}$$

∴ then Amplitude of (z_1+z_2) is $k\pi$

$\therefore k\in\left]\frac{-1}{2},\frac{1}{2}\right[$ $\quad$ ∴ the Amplitude $\in\left]\frac{-\pi}{2},\frac{\pi}{2}\right[$

∴ the Amplitude could be equal $\frac{-\pi}{6}$

7 (c)

Solution :

$\frac{3}{3}=\frac{12}{12}=\frac{-4}{-4}\neq\frac{9}{-17}$

∴ the two planes are parallel

put : $y=0$, $z=0$ in the equation of the 1st plane

$\therefore x=3$

∴ the point be $(3,0,0)$ lying in the 1st plane.

∴ the distance between the two planes

$=\frac{|3(3)+12(0)-4(0)+17|}{\sqrt{9+144+16}}=2$ length unit

8 (a)

Solution :

The direction vector $\vec{d}=\overrightarrow{BA}=(2,-1,1)$

, the straight line passes through the point $A(1,-1,2)$

∴ equation of the straight line is : $\frac{x-1}{2}=\frac{y+1}{-1}=\frac{z-2}{1}$

9 (b)

Solution :

$\because (1+ax)^n=1+{}^nc_1(ax)+{}^nc_2(ax)^2+\dots+(ax)^n$

$=1+nax+\frac{n(n-1)}{2}a^2x^2+\dots+a^nx^n$

$\because (1+ax)^n=1+30x+405x^2+\dots+a^nx^n$

$\therefore na=30$ $\quad$ (1)

then $a=\frac{30}{n}$, $\frac{n(n-1)}{2}a^2=405$

$\therefore \frac{n(n-1)}{2}\left(\frac{30}{n}\right)^2=405$ $\quad\therefore \frac{n-1}{n}=\frac{9}{10}$

$\therefore 10n-10=9n$ $\quad\therefore n=10$

from (1) : $\therefore a=3$

$\therefore n:a=10:3$

10 (c)

Solution :

$z=(1+2\cos^2 20^\circ-1+i\times 2\sin 20^\circ\cos 20^\circ)$

$=2\cos 20^\circ(\cos 20^\circ+i\sin 20^\circ)$

amplitude of $(z)=\frac{\pi}{9}$

11 (d)

Solution :

∵ The normal direction vector to the given plane is $(2,3,-5)$

∵ The two planes are parallel

∴ The normal direction vector of the required plane is $(2,3,-5)$

∵ The plane passes through the point $(-2,2,-1)$

∴ its equation is

$(2,3,-5)\cdot\vec{r}=(2,3,-5)\cdot(-2,2,-1)$

$\therefore 2x+3y-5z=7$

12 (c)

Solution :

$\because {}^nP_r={}^nP_{r+1}$ $\quad\therefore \frac{\lfloor n}{\lfloor n-r}=\frac{\lfloor n}{\lfloor n-r-1}$

$\therefore \frac{1}{n-r}=1$ $\quad\therefore n-r=1$ $\quad$ (1)

$\because {}^nC_r={}^nC_{r-1}$ $\quad\therefore \frac{{}^nC_r}{{}^nC_{r-1}}=1$

$\therefore \frac{n-r+1}{r}=1$ $\quad\therefore n-r+1=r$

$\therefore n-2r=-1$ $\quad$ (2)

from (1) , (2) : $\therefore r=2$, $n=3$ $\quad\therefore n+r=5$

13 (a)

Solution :

The unit vector in direction of vector $\vec{A}$ is $\vec{u}_A=(\cos 135^\circ,\cos 60^\circ,\cos 120^\circ)$

$=\left(\frac{-\sqrt{2}}{2},\frac{1}{2},\frac{-1}{2}\right)$

$\therefore \vec{A}=\|\vec{A}\|\,\vec{u}_A=2\left(\frac{-\sqrt{2}}{2},\frac{1}{2},\frac{-1}{2}\right)$

$=(-\sqrt{2},1,-1)$

$$\because \vec{A}\cdot\vec{B}\times\vec{C}=\begin{vmatrix}-\sqrt{2} & 1 & -1\\ 1 & \sqrt{2} & 0\\ \sqrt{2} & 3 & 5\end{vmatrix}=-\sqrt{2}\left(5\sqrt{2}-0\right)-1(5-0)-1(3-2)=-16$$

∴ Volume of the parallelepiped $=|-16|$

$=16$ cubic unit.

14 (a)

Solution :

The component of the vector $\vec{B}$ in direction vector $\vec{A}=\frac{\vec{A}\cdot\vec{B}}{\|\vec{A}\|}$

$\therefore \frac{\vec{A}\cdot\vec{B}}{5}=3$ $\quad\therefore \vec{A}\cdot\vec{B}=15$

15 (c)

Solution :

∵ $(x-3)$ is a factor of the determinant.

∴ $x=3$ makes value of the determinant $=0$

$$\therefore \begin{vmatrix}1 & m & 3\\ 1 & 5 & 3\\ 2 & 2 & 4\end{vmatrix}=0$$

$\therefore 1\times 14-m\times -2+3\times -8=0$

$\therefore m=5$

16 (d)

17 (a)

Solution :

$\because \vec{n_1}=(3,-4,2)$, $\vec{n_2}=(3,4,-m)$

∵ The two planes are perpendicular.

$\therefore \vec{n_1}\cdot\vec{n_2}=0$ $\quad\therefore (3,-4,2)\cdot(3,4,-m)=0$

$\therefore 9-16-2m=0$, $m=\frac{-7}{2}$

18 (b)

Solution :

∵ The coefficient of $T_6={}^{10}C_5$

$\therefore {}^{10}C_5\times\left(\frac{1}{b}\right)^5(a)^5={}^{10}C_5$

$\therefore \left(\frac{a}{b}\right)^5=1$ $\quad\therefore \frac{a}{b}=1$

19 (b)

Solution :

$\because \frac{{}^nC_4+{}^nC_3}{{}^{n+1}C_3}=\frac{{}^{n+1}C_4}{{}^{n+1}C_3}=\frac{(n+1)-4+1}{4}=\frac{n-2}{4}$

$\therefore \frac{n-2}{4}=1$ $\quad\therefore n=6$

$\therefore \lfloor n-6=\lfloor 6-6=1$

20 (d)

Solution :

$\because \cos\theta_x=\frac{A_x}{\|\vec{A}\|}$ $\quad\therefore \frac{3}{13}=\frac{K}{\sqrt{K^2+(12)^2+(4)^2}}$

$\therefore 9(k^2+160)=169k^2$ $\quad\therefore 9k^2+9\times 160=169k^2$

$\therefore 160k^2=9\times 160$ $\quad\therefore k^2=9$

$\therefore k=-3$ (refused because the angle is acute)

or $k=3$

21 (c)

Solution :

$\because (x-1)(x^2+x+1)=8 \qquad \therefore x^3-1=8$

$\therefore x^3=9$

$\therefore x^9+1=(9)^3+1=730$

---

22 (c)

Solution :

$\because A=\begin{pmatrix}1&4&0\\0&a&1\\-1&0&a\end{pmatrix}, A^*=\begin{pmatrix}1&4&0&:5\\0&a&1&:3\\-1&0&a&:1\end{pmatrix}$

$\because$ The system of the equation has infinite number of solutions.

$\therefore Rk(A)=RK(A^*)<3$

$\therefore \begin{vmatrix}1&4&0\\0&a&1\\-1&0&a\end{vmatrix}=0 \quad \therefore 1(a^2-0)-4(0+1)=0$

$\therefore a^2-4=0$

$\therefore a=-2$ (refused) because it makes $RK(A^*)=3$ or $a=2$

---

23 (b)

Solution :

Let D is midpoint of $\overline{BC}$

$\because \overrightarrow{AB}+\overrightarrow{AC}=2\,\overrightarrow{AD}$

$\therefore 2\,\overrightarrow{AD}=(3,0,-3)+(1,-2,1)=(4,-2,-2)$

$\therefore \overrightarrow{AD}=(2,-1,-1)$

length of $\overline{AD}=\|\overrightarrow{AD}\|=\sqrt{4+1+1}=\sqrt{6}$ length unit

---

24 (a)

Solution :

$\because A=\begin{pmatrix}1&1&0\\0&1&1\\1&0&1\end{pmatrix} \qquad \therefore |A|=2$

Cofactors matrix $=\begin{pmatrix}1&1&-1\\-1&1&1\\1&-1&1\end{pmatrix}$

$\therefore Adj(A)=\begin{pmatrix}1&-1&1\\1&1&-1\\-1&1&1\end{pmatrix}$

$\therefore A^{-1}=\frac{1}{2}\begin{pmatrix}1&-1&1\\1&1&-1\\-1&1&1\end{pmatrix}$

$\begin{pmatrix}x\\y\\z\end{pmatrix}=\frac{1}{2}\begin{pmatrix}1&-1&1\\1&1&-1\\-1&1&1\end{pmatrix}\times\begin{pmatrix}3\\4\\5\end{pmatrix}=\begin{pmatrix}2\\1\\3\end{pmatrix}$

$\therefore x=2 \;,\; y=1 \;,\; z=3$

$\therefore \frac{1}{x}+\frac{1}{y}+\frac{1}{z}=\frac{1}{2}+1+\frac{1}{3}=\frac{11}{6}$

---

25 (d)

Solution :

$T_{r+1}={}^nC_r\left(\frac{5}{x}\right)^r(x^3)^{n-r}={}^nC_r\times 5^r x^{3n-4r}$

$\because$ The term free of x is T_7

$\therefore r=6 \qquad \therefore 3n-24=0 \qquad \therefore n=8$

Exam 5

1 (a)

Solution :

$\sum\limits_{r=0}^{n}\frac{{}^nP_r}{\underline{|r}}=\sum\limits_{r=0}^{n}{}^nC_r={}^nC_0+{}^nC_1+{}^nC_2+\ldots+{}^nC_n=2^n$

---

2 (d)

Solution :

$\because$ volume of the sphere $=\frac{4}{3}\pi r^3=36\pi$

$\therefore r^3=27$, then $r=3$ length unit.

$\therefore$ equation of the sphere is $(x+1)^2+y^2+(z-5)^2=9$

---

3 (b)

Solution :

$\overrightarrow{d_1}=(2,-1,1)\,,\,\overrightarrow{d_2}=(a,-2,-1)$

$\cos 60^\circ=\frac{|(2,-1,1).(a,-2,-1)|}{\sqrt{4+1+1}\times\sqrt{a^2+4+1}}$

$\therefore \frac{1}{2}=\frac{|2a+1|}{\sqrt{6}\sqrt{a^2+5}}$

$\therefore 4(2a+1)^2=6(a^2+5)$

$\therefore 16a^2+16a+4=6a^2+30$

$\therefore 5a^2+8a-13=0$

$\therefore (5a+13)(a-1)=0$

$\therefore a=\frac{-13}{5}$ (refused) or $a=1$

---

4 (a)

Solution :

$\because A=\begin{pmatrix}0&0&-1\\0&-1&0\\-1&0&0\end{pmatrix} \qquad \therefore |a|=1$

$\therefore$ Cofactors matrix $=\begin{pmatrix}0&0&-1\\0&-1&0\\-1&0&0\end{pmatrix}$

$\therefore Adj(A)=\begin{pmatrix}0&0&-1\\0&-1&0\\-1&0&0\end{pmatrix}$

$\therefore A^{-1}=\begin{pmatrix}0&0&-1\\0&-1&0\\-1&0&0\end{pmatrix}$

$\therefore A^{-2}=(A^{-1})^2=\begin{pmatrix}0&0&-1\\0&-1&0\\-1&0&0\end{pmatrix}\begin{pmatrix}0&0&-1\\0&-1&0\\-1&0&0\end{pmatrix}$

$=\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}=I$

---

5 (b)

Solution :

$\because a={}^nP_2 \qquad \therefore a=n(n-1) \qquad (1)$

$\because {}^aC_2=\frac{{}^aP_2}{\underline{|2}}=\frac{a(a-1)}{\underline{|2}}$

$\therefore {}^aC_2=\frac{\left(n(n-1)\right)\left(n(n-1)-1\right)}{\underline{|2}}=\frac{{}^nP_2}{\underline{|2}}\times(n^2-n-1)$

$\therefore {}^aC_2={}^nC_2(n^2-n-1)$

---

6 (a)

Solution :

By selecting two different numbers from $\{1,2,3,4,5,7,8,9\}$ we have ${}^8P_2$ ways and Putting number. "6" in the three places

$\therefore$ The number of ways to form secrete number $=3\times{}^8P_2=168$ ways

---

7 (c)

Solution :

$32(\cos\theta+i\sin\theta)^2=\left(1+\sqrt{3}\,i\right)^5$

$\therefore 32(\cos 2\theta+i\sin 2\theta)=\left[2\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)\right]^5$

$\therefore \cos 2\theta+i\sin 2\theta=\cos\frac{5\pi}{3}+i\sin\frac{5\pi}{3}$

$\therefore 2\theta=\frac{5\pi}{3} \qquad \therefore \theta=\frac{5\pi}{6}$

---

8 (a)

Solution :

$\theta_z=90^\circ-\ell \;,\; \theta_x=90^\circ-m \;,\; \theta_y=90^\circ-n$

$\because \cos^2\theta_x+\cos^2\theta_y+\cos^2\theta_z=1$

$\therefore \cos^2(90^\circ-m)+\cos^2(90^\circ-n)+\cos^2(90^\circ-\ell)=1$

$\therefore \sin^2 m+\sin^2 n+\sin^2\ell=1$

---

9 (a)

Solution :

$\because$ The two straight lines are perpendicular

$\therefore \overrightarrow{d_1}.\overrightarrow{d_2}=$ zero

$\therefore (-1,3,4).(m,n,1)=0$

$\therefore -m+3n+4=0 \qquad \therefore 3n-m=-4$

---

10 (c)

Solution :

Let (x_1,y_1,z_1) are any point on the required plane.

$\because$ The required plane bisects the angle between the two planes.

$P_1: 2x-y+2z+3=0$

$, P_2: 3x-2y+6z+8=0$

$\therefore$ Perpendicular distant from (x_1,y_1,z_1) to each of the two planes are equal

$\frac{|2x_1-y_1+2z_1+3|}{\sqrt{4+1+4}}=\frac{|3x_1-2y_1+6z_1+8|}{\sqrt{9+4+36}}$

$\therefore 7(2x_1-y_1+2z_1+3)=\pm 3(3x_1-2y_1+6z_1+8)$

$\therefore 5x-y-4z-3=0$

or $23x - 13y + 32z + 45 = 0$

i.e. : there exist two perpendicular planes one of them bisects the acute angle and the other bisect the obtuse angle between the two planes p_1 , p_2

---

11 (a)

Solution :

Centre of the sphere $M = (2, 1, -3)$

$\because$ The normal direction vector to the plane

$\vec{n} = (2, -1, 2)$

$\therefore$ direction vector of the straight line $\vec{d}$

$= \vec{n} = (2, -1, 2)$

$\therefore$ The equation of the straight line is :

$\vec{r} = (2, 1, -3) + t(2, -1, 2)$

---

12 (b)

Solution :

$$\because f(x) = \begin{vmatrix} 1 & 2 & 3 \\ 0 & x & 5 \\ 0 & 2 & x+3 \end{vmatrix} \text{"by doing } R_3 - R_1\text{"}$$

$$= \begin{vmatrix} 1 & 2 & 3 \\ 0 & x & 5 \\ 0 & 0 & x \end{vmatrix} = x^2$$

$\therefore f(1) + f(2) + f(3) + \dots + f(10)$

$= 1^2 + 2^2 + 3^2 + \dots + 10^2$

$$= \sum_{r=1}^{10} r^2 = \frac{10 \times 11 \times 21}{6} = 385$$

---

13 (a)

Solution :

$$T_7 = {}^{12}C_6 \times \left(\frac{-b}{x}\right)^6 \times (a x^2)^6 = {}^{12}C_6 \times (a b)^6 \times x^6$$

$\therefore T_7$ is the term contains x^6

---

14 (d)

Solution :

$$\sqrt{2(\omega + i)(\omega^2 + i)} = \sqrt{2\left(\omega^3 + i(\omega + \omega^2) + i^2\right)}$$

$$= \sqrt{2(1 - i + (-1))} = \sqrt{-2i}$$

$$= \sqrt{(1-i)^2} = \pm(1 - i)$$

15 (b)

Solution :

$$\because m = \begin{vmatrix} 1 & x & y \\ x & 1 & y \\ y & x & 1 \end{vmatrix} \text{"by doing } C_1 + (C_2 + C_3)\text{"}$$

$$= \begin{vmatrix} 1+x+y & x & y \\ 1+x+y & 1 & y \\ 1+x+y & x & 1 \end{vmatrix}$$

$$= (1+x+y) \begin{vmatrix} 1 & x & y \\ 1 & 1 & y \\ 1 & x & 1 \end{vmatrix} \text{"by doing } R_3 - R_1 \text{, } R_2 - R_1\text{"}$$

$$= (1+x+y) \begin{vmatrix} 1 & x & y \\ 0 & 1-x & 0 \\ 0 & 0 & 1-y \end{vmatrix}$$

$= (1+x+y)\ (1-x)(1-y)$ "Put $x + y = 2$"

$\therefore m = 3(1-x)(x-1) = -3(1-x)^2$

$\therefore \grave{m} = 6(1-x)$

$\therefore \grave{m} = 0$ when $1 - x = 0$ $\qquad \therefore x = 1$

, $\ddot{m} = -6 < 0$ for all values of x

$\therefore m$ has maximum value at $x = 1$

$\therefore$ The maximum value $= -3(1-1)^2 =$ zero

---

16 (a)

Solution :

$$\sqrt{(2+4)^2 + (k-4)^2 + (3-2)^2} = \sqrt{62}$$

$\therefore 36 + k^2 - 8k + 16 + 1 = 62$

$\therefore k^2 - 8k - 9 = 0 \qquad \therefore (k-9)(k+1) = 0$

$\therefore k = 9$ or $k = -1$

---

17 (a)

Solution :

$$\because \text{matrix of coefficients } (A) = \begin{pmatrix} 1 & -1 & 1 \\ 2 & 1 & -1 \\ 4 & 1 & 1 \end{pmatrix}$$

$$\because |A| = \begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & -1 \\ 4 & 1 & 1 \end{vmatrix} = 1(1+1) + 1(2+4) + 1(2-4)$$

$= 6$

$\therefore Rk(A) = 3$

$$\because A^* = \begin{pmatrix} 1 & -1 & 1 : 2 \\ 2 & 1 & -1 : 2 \\ 4 & 1 & 1 : 0 \end{pmatrix}$$

$\therefore Rk(A) = Rk(A^*) =$ number of unknowns $= 3$

$\therefore$ The system has a unique solution

---

18 (a)

Solution :

Coefficient of $T_9 = {}^{12}C_8 \times \left(\frac{-1}{a}\right)^8 \times a^4 = {}^{12}C_8 \times a^{-4}$

$\therefore {}^{12}C_8 \times a^{-4} = 7920$

$\therefore a^4 = \frac{1}{16} \qquad \therefore a = \pm\frac{1}{2}$

---

19 (a)

Solution :

$$T_{r+1} = {}^{15}C_r \times \left(\frac{-1}{x}\right)^r \times (x^2)^{15-r}$$

$$= {}^{15}C_r \times (-1)^r \times x^{30-3r}$$

Put $30 - 3r = 0 \qquad \therefore r = 10$

$\therefore T_{11}$ is the term free of x

$\therefore T_{11} = {}^{15}C_{10} \times (-1)^{10} = {}^{15}C_{10} = {}^{15}C_5$

---

20 (d)

Solution :

$\because$ D is the midpoint of $\overline{BC} = \left(\frac{2+0}{2}, \frac{3+3}{2}, \frac{7+1}{2}\right)$

$= (1, 3, 4)$

length of $\overline{AD} = \sqrt{(3-1)^2 + (1-3)^2 + (5-4)^2}$

$= \sqrt{4 + 4 + 1} = 3$ length unit

---

21 (c)

Solution :

$$\frac{1+a}{1-a} = \frac{1 + \cos\theta + i\sin\theta}{1 - \cos\theta - i\sin\theta}$$

$$= \frac{1 + 2\cos^2\frac{\theta}{2} - 1 + i \times 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{1 - 1 + 2\sin^2\frac{\theta}{2} - i \times 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}$$

$$= \frac{2\cos\frac{\theta}{2}\left(\cos\frac{\theta}{2} + i\sin\frac{\theta}{2}\right)}{2\sin\frac{\theta}{2}\left(\sin\frac{\theta}{2} - i\cos\frac{\theta}{2}\right)}$$

$$= \frac{\cot\frac{\theta}{2}\left(\cos\frac{\theta}{2} + i\sin\frac{\theta}{2}\right)}{\cos\left(\frac{\theta}{2} - 90^\circ\right) + i\sin\left(\frac{\theta}{2} - 90^\circ\right)}$$

$$= \cot\frac{\theta}{2} \times (\cos 90^\circ + i\sin 90^\circ) = i\cot\frac{\theta}{2}$$

---

22 (a)

Solution :

The geometric centre of the triangle

$M = \left(\frac{3+15+0}{3}, \frac{-4+0-8}{3}, \frac{0+2+4}{3}\right) = (6, -4, 2)$

the distance of the centre of the triangle from the plane $xy = |2| = 2$ and from $xz = |-4| = 4$

$\therefore$ the distance from xz is the greater than distance from xy plane

---

23 (a)

Solution :

(by taking 7^n common factor from C_3)

$$\therefore \Delta = 7^n \begin{vmatrix} 7^{2n} & 7^{3n} & 7^{3n} \\ 7^{3n} & 7^{4n} & 7^{4n} \\ 7^{4n} & 7^{5n} & 7^{5n} \end{vmatrix}$$

$\because C_2 = C_3 \qquad \therefore \Delta = 0$

---

24 (a)

Solution :

$a e^{2\theta i} + b e^{-\theta i}$

$= a(\cos 2\theta + i\sin 2\theta) + b\left(\cos(-2\theta) + i\sin(-2\theta)\right)$

$= a\cos 2\theta + a\,i\sin 2\theta + b\cos 2\theta - b\,i\sin 2\theta$

$= (a+b)\cos 2\theta + (a-b)\,i\sin 2\theta$

$\therefore (a+b)\cos 2\theta + (a-b)\,i\sin 2\theta = 5\cos 2\theta - i\sin 2\theta$

$\therefore a + b = 5 \quad (1) \qquad , \qquad a - b = -1 \quad (2)$

from (1) , (2) we get $a = 2$, $b = 3 \quad \therefore ab = 6$

---

25 (a)

Solution :

$\because bcx + acy + abz = abc$

$\therefore \frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$

$\therefore K(a, 0, 0)$, $N(0, b, 0)$

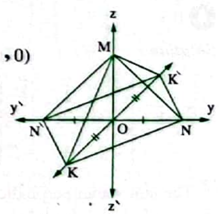
$M(0, 0, c)$

$\because bcx + acy - abz$

$= -abc$

$\therefore \frac{x}{-a} + \frac{y}{-b} + \frac{z}{c} = 1$

$\therefore \grave{K}(-a, 0, 0)$, $\grave{N}(0, -b, 0)$



From the graph we get : $\overline{MO} \perp$ plane $KN\grave{K}\grave{N}$

, $ON = O\grave{N}$, $OK = O\grave{K}$, $\overline{N\grave{N}} \perp \overline{K\grave{K}}$

$\therefore$ The two diagonals bisect

, each other and are perpendicular

$\because a \neq b$ $\qquad \therefore K\grave{K} \neq N\grave{N}$

$\therefore$ $KN\grave{K}\grave{N}$ is a rhombus

$\therefore$ $MKN\grave{K}\grave{N}$ is a right pyramid.

Exam 6

1 (d)

Solution :

$\cos^2 30° + \cos^2 70° + \cos^2 \theta = 1$

$\therefore \cos\theta = 0.365 \qquad \therefore \theta = 68.6°$

2 (b)

Solution :

$e^{\pi i} - e^{-\pi i} = (\cos\pi + i\sin\pi) - (\cos(-\pi) + i\sin(-\pi))$

$= (-1+0) - (-1+0) = 0$

3 (d)

Solution :

$^{47}C_4 + \sum_{r=1}^{5} {}^{52-r}C_3$

$= {}^{47}C_4 + {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{48}C_3 + {}^{47}C_3$

$= ({}^{47}C_4 + {}^{47}C_3) + {}^{48}C_3 + {}^{49}C_3 + {}^{50}C_3 + {}^{51}C_3$

$= ({}^{48}C_4 + {}^{48}C_3) + {}^{49}C_3 + {}^{50}C_3 + {}^{51}C_3$

$= ({}^{49}C_4 + {}^{49}C_3) + {}^{50}C_3 + {}^{51}C_3 = ({}^{50}C_4 + {}^{50}C_3) + {}^{51}C_3$

$= {}^{51}C_4 + {}^{51}C_3 = {}^{52}C_4$

4 (b)

Solution :

$\because \vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -1 & 3 \\ -2 & 1 & -2 \end{vmatrix} = -\hat{i} + 2\hat{j} + 2\hat{k}$

$\therefore$ The unit vector perpindicular to each $\vec{a}$, $\vec{b}$

$= \pm \frac{-\hat{i}+2\hat{j}+2\hat{k}}{\sqrt{1+4+4}} = \pm \frac{1}{3}(-\hat{i}+2\hat{j}+2\hat{k})$

5 (a)

Solution :

The normal direction vector of the plane.

$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 6 & -2 & -1 \\ 1 & -3 & 3 \end{vmatrix} = (-9 , -19 , -16)$

$\because$ The plane contains the st. line

$\vec{r} = (0 , 3 , -5) + t_1 (6 , -2 , -1)$

$\therefore$ The plane contain the point $(0 , 3 , -5)$

$\therefore$ The equation is

$\vec{r} . (-9 , -19 , -16) = (0 , 3 , -5) . (-9 , -19 , -16)$

$\therefore -9x - 19y - 16z - 23 = 0$

i.e. $9x + 19y + 16z + 23 = 0$

6 (c)

Solution :

The coordinates of the point A $(0 , 3 , 2)$

, coordinates of the point B $(2 , 0 , 8)$

, coordinates of the point C $(2 , 0 , 4)$

$\therefore \overrightarrow{BA} = (-2 , 3 , -6)$, $\overrightarrow{BC} = (0 , 0 , -4)$

$\cos\theta = \frac{(-2 , 3 , -6) . (0 , 0 , -4)}{\sqrt{4+9+36} \times \sqrt{(-4)^2}} = \frac{6}{7}$

$\therefore \theta = 31°$

7 (b)

Solution :

$\frac{8n+2}{2} = 9 \qquad \therefore 8n = 16 \qquad \therefore n = 2$

8 (b)

Solution :

Number of ways of the answer $= {}^5C_4 \times {}^8C_6 + {}^5C_5 \times {}^8C_5$

$= 196$

9 (b)

Solution :

$A = \begin{pmatrix} 2 & 2 & 3 \\ 1 & -2 & -3 \\ 4 & -2 & -3 \end{pmatrix}$

$\therefore |A| = \begin{vmatrix} 2 & 2 & 3 \\ 1 & -2 & -3 \\ 4 & -2 & -3 \end{vmatrix} = 0$

$\therefore Rk(A) < 3 \qquad , \because \begin{vmatrix} 2 & 2 \\ 1 & -2 \end{vmatrix} = -4 - 2 = -6 \neq 0$

$\therefore Rk(A) = 2 \qquad \therefore Rk(A) <$ Number of unknow

$\therefore$ The system has infinite number of solutions one of them is trivial solution.

10 (b)

Solution :

$\because$ The direction vector of the given straight line is $(1 , -1 , 1)$

, the normal direction of the plane is $(2 , -1 , 1)$

$\therefore$ Measure of the angle between the straight line and the plane $= 90 - \theta$ where :

$\cos\theta = \frac{|(1 , -1 , 1) . (2 , -1 , 1)|}{\sqrt{1+1+1} \times \sqrt{4+1+1}} \qquad \therefore \theta = 19° \, 2\grave{8}$

$\therefore$ Measure of the angle between the st. line and the plane $= 70° \, 3\grave{2}$

11 (c)

Solution :

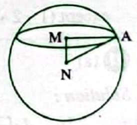
$\because$ The two straight lines are perpendicular

$\therefore (2 , -1 , m) . (m , 1 , -1) = 0$

$\therefore 2m - 1 - m = 0 \qquad \therefore m = 1$

12 (c)

Solution :



Centre of the sphere $(0 , 1 , 2)$

(radius of the sphere) = NA

$= \sqrt{0+1+4+20} = 5$ length unit

length of $\overline{MN} = \frac{|0 + 2 \times 1 + 2 \times 2 - 15|}{\sqrt{1+4+4}}$

$= 3$ length unit

$\therefore$ (radius of the circular corss section) MA

$= \sqrt{5^2 - 3^2} = 4$ length unit.

13 (c)

Solution :

$T_{r+1} = {}^6C_r (x^{-1})^r \times (x^5)^{6-r} = {}^6C_r \times x^{30-6r}$

Put $30 - 6r = 0$

$\therefore r = 5$

T_6 is the term free of x

$\therefore$ order of the middle term $= \frac{6+2}{2} = 4$

$\therefore$ The ratio between the term free of x and coeffecient of the middle term $= \frac{T_6}{\text{coeff.}T_4} = \frac{{}^6C_5}{{}^6C_3} = \frac{6}{20} = \frac{3}{10}$

14 (c)

Solution :

$\because \begin{vmatrix} y+z & x & x \\ y & z+x & y \\ z & z & x+y \end{vmatrix}$ (by doing $R_1 - R_3$)

$= \begin{vmatrix} y & x-z & -y \\ y & z+x & y \\ z & z & x+y \end{vmatrix}$ (by doing $R_1 + R_2$)

$= \begin{vmatrix} 2y & 2x & 0 \\ y & z+x & y \\ z & z & x+y \end{vmatrix}$

$= 2\begin{vmatrix} y & x & 0 \\ y & z+x & y \\ z & z & x+y \end{vmatrix}$ (by doing $R_2 - R_1$)

$= 2\begin{vmatrix} y & x & 0 \\ 0 & z & y \\ z & z & x+y \end{vmatrix}$ (by doing $R_3 - R_2$)

$= 2\begin{vmatrix} y & x & 0 \\ 0 & z & y \\ z & 0 & x \end{vmatrix}$

15 (b)

Solution :

The matrix equation is

$\begin{pmatrix} 2 & -1 & -3 \\ 1 & 2 & 1 \\ 3 & -5 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 13 \end{pmatrix}$

$\therefore |A| = \begin{vmatrix} 2 & -1 & -3 \\ 1 & 2 & 1 \\ 3 & -5 & 2 \end{vmatrix} = 50 \neq 0$

$\because Adj(A) = \begin{pmatrix} 9 & 1 & -11 \\ 17 & 13 & 7 \\ 5 & -5 & 5 \end{pmatrix}^t = \begin{pmatrix} 9 & 17 & 5 \\ 1 & 13 & -5 \\ -11 & 7 & 5 \end{pmatrix}$

$$\therefore A^{-1} = \frac{1}{|A|} \text{Adj}(A) = \frac{1}{50}\begin{pmatrix} 9 & 17 & 5 \\ 1 & 13 & -5 \\ -11 & 7 & 5 \end{pmatrix}$$

$$\therefore \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{50}\begin{pmatrix} 9 & 17 & 5 \\ 1 & 13 & -5 \\ -11 & 7 & 5 \end{pmatrix}\begin{pmatrix} 2 \\ 1 \\ 13 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$$

$x = 2$, $y = -1$, $z = 1$

$\therefore x + y + z = 2$

16 (d)

Solution :

$\because z_1 = 1 - \sqrt{3}\,i$ $\qquad \therefore r = \sqrt{1^2 + (-\sqrt{3})^2} = 2$

$\because x > 0$, $y < 0$ $\qquad \therefore \theta$ lies in the 4^{th} quad.

$\therefore \theta = \tan^{-1}\left(\frac{-\sqrt{3}}{1}\right) = -60° = \frac{-\pi}{3}^{rad}$

$\therefore z_1 = 2e^{\frac{-\pi}{3}i}$

$\because z_2 = 1 + i$

$\therefore r = \sqrt{1^2 + 1^2} = \sqrt{2}$

$\because x > 0$, $y > 0$ $\qquad \therefore \theta$ lies in 1^{st} quad.

$\theta = \tan^{-1}\left(\frac{1}{1}\right) = 45° = \frac{\pi}{4}^{rad}$

$\therefore z_2 = \sqrt{2}\,e^{\frac{\pi}{3}i}$

$\therefore z = \frac{z_1}{z_2} = \frac{2e^{\frac{-\pi}{3}i}}{\sqrt{2}\,e^{\frac{-\pi}{4}i}} = \sqrt{2}\,e^{\frac{-7}{12}\pi i}$

17 (c)

Solution :

$\because z = 2\sqrt{2}(1 + i) = 2\sqrt{2} + 2\sqrt{2}\,i$

$\therefore r = \sqrt{(2\sqrt{2})^2 + (2\sqrt{2})^2} = 4$

$\because x > 0$, $y > 0$

$\therefore \theta$ lies in the 1^{st} quad.

$\therefore \theta = \tan^{-1}\left(\frac{2\sqrt{2}}{2\sqrt{2}}\right) = 45° = \frac{\pi}{4}^{rad}$

$\therefore z = \left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

18 (d)

Solution :

$2\|\vec{A} \times \vec{B}\|(\vec{A} \times \vec{B})$

$= 2\left(\|\vec{A}\|\|\vec{B}\|\sin\theta\right) \times \left(\|\vec{A}\|\|\vec{B}\|\cos\theta\right)$

$= 2(1 \times 1 \times \sin\theta)(1 \times 1 \times \cos\theta)$

$= 2\sin\theta\cos\theta = \sin 2\theta$

19 (d)

Solution :

${}^nC_r = {}^nP_r$ $\qquad \therefore \frac{{}^nP_r}{\lfloor r} = {}^nP_r$

$\therefore \lfloor r = 1$ $\qquad \therefore r = 0$ or 1

20 (a)

Solution :

Put $\vec{r} = (1, 2, 1)$

$\therefore (1, 2, 1) \cdot (3, -5, 4) = 3 - 10 + 4 = -3 < 5$

Put $\vec{r} = (0, -1, 2)$

$\therefore (0, -1, 2) \cdot (3, -5, 4) = 0 + 5 + 8 = 13 > 5$

Put $\vec{r} = (2, -3, 1)$

$\therefore (2, -3, 1) \cdot (3, -5, 4) = 6 + 15 + 4 = 25 > 5$

Put $\vec{r} = (1, 2, 4)$

$\therefore (1, 2, 4) \cdot (3, -5, 4) = 3 - 10 + 16 = 9 > 5$

$\therefore$ all of their points lying on same side except $(1, 2, 1)$

21 (a)

Solution :

$T_{r+1} = {}^7C_r \times (\sqrt{3})^{-r} \times (\sqrt{3})^{7-r}$

$= {}^7C_r \times (\sqrt{3})^{7-2r} = {}^7C_r \times 3^{3\frac{1}{2} - r}$

$\because 3\frac{1}{2} - r \notin N$, $r = 0, 1, 2, \ldots, 7$

$\therefore$ Number of integer solution = zero

22 (d)

Solution :

$\left(\frac{\omega}{1 + 2\omega}\right)^2 + \left(\frac{\omega^2}{1 + 2\omega}\right) = \frac{\omega^2}{(1 + 2\omega)^2} + \frac{\omega^4}{(1 + 2\omega)^2}$

$= \frac{\omega^2 + \omega}{(1 + 2\omega)^2} = \frac{-1}{1 + 4\omega + 4\omega^2}$

$= \frac{-1}{1 - 4} = \frac{1}{3}$

23 (b)

Solution :

$\overrightarrow{AB} = (1, 0, 2) . \overrightarrow{AC} = (2, 1, 1) . \overrightarrow{AD} = (2, 2, 3)$

$\overrightarrow{AB} \cdot \overrightarrow{AC} \times \overrightarrow{AD} = \begin{vmatrix} 1 & 0 & 2 \\ 2 & 1 & 1 \\ 2 & 2 & 3 \end{vmatrix}$

$= 1(3 - 2) + 2(4 - 2) = 5$

$\therefore$ volume of parallel piped $= |\overrightarrow{AB} \cdot \overrightarrow{AC} \times \overrightarrow{AD}|$

$= |5| = 5$ cubic unit.

24 (c)

Solution :

$|A| = \begin{vmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{vmatrix} = a^3$

$\therefore A^{adj} = \begin{pmatrix} a^2 & 0 & 0 \\ 0 & a^2 & 0 \\ 0 & 0 & a^2 \end{pmatrix}$

$\therefore |A^{adj}| = a^6$ $\qquad \therefore |A||A^{adj}| = a^3 \times a^6 = a^9$

25 (b)

Solution :

$\begin{vmatrix} a^3 - 1 & a^2 & a \\ b^3 - 1 & b^2 & b \\ c^3 - 1 & c^2 & c \end{vmatrix} = 0$

$\begin{vmatrix} a^3 & a^2 & a \\ b^3 & b^2 & b \\ c^3 & c^2 & c \end{vmatrix} - \begin{vmatrix} 1 & a^2 & a \\ 1 & b^2 & b \\ 1 & c^2 & c \end{vmatrix} = 0$

$\therefore abc\begin{vmatrix} a^2 & a & 1 \\ b^2 & b & 1 \\ c^2 & c & 1 \end{vmatrix} + \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = 0$

$abc\begin{vmatrix} a^2 & a & 1 \\ b^2 & b & 1 \\ c^2 & c & 1 \end{vmatrix} - \begin{vmatrix} a^2 & a & 1 \\ b^2 & b & 1 \\ c^2 & c & 1 \end{vmatrix} = 0$

$(abc - 1)\begin{vmatrix} a^2 & a & 1 \\ b^2 & b & 1 \\ c^2 & c & 1 \end{vmatrix} = 0$

in case $a \neq b \neq c$ $\qquad \therefore \Delta \neq 0$

$\therefore abc - 1 = 0$ $\qquad \therefore abc = 1$

Exam 7

1 (c)

2 (d)

Solution :

Put $x = 1$

$\therefore$ sum of coeffecients of the terms $= \left(1 - \frac{1}{1}\right)^7 =$ zero

3 (c)

Solution :

The modulus = 3 , the amplitude = 90 + 30 = 120°

$\therefore$ the complex number = 3 (cos 120 + i sin 120)

4 (b)

Solution :

$\because$ The plane is perpendicular to the st. line which passes through the two points

$(-3, 1, 2), (2, 3, 4)$

$\therefore$ The vector $= (-3, 1, 2) - (2, 3, 4)$

$= (-5, -2, -2)$ is normal direction of the plane.

$\therefore$ Equation of the plane is

$\vec{r} \cdot (-5, -2, -2) = (-1, 2, 1) \cdot (-5, -2, -2)$

$\therefore -5x - 2y - 2z + 1 = 0$

5 (c)

Solution :

$\because \frac{4}{1} \neq \frac{1}{-1} \neq \frac{3}{2}$

$\therefore$ The two st. lines are not parallel.

At the point of intersection of two st. lines $\therefore \vec{r_1} = \vec{r_2}$

$\therefore (3, -1, 2) + t_1(4, 1, 3)$

$= (0, 4, -1) + t_2(1, -1, 2)$

$\therefore 3 + 4t_1 = t_2$ $\qquad \therefore 4t_1 - t_2 = -3$ (1)

$, -1 + t_1 = 4 - t_2$ $\qquad \therefore t_1 + t_2 = 5$ (2)

$, 2 + 3t_1 = -1 + 2t_2$ $\qquad \therefore 3t_1 - 2t_2 = -3$ (3)

From (1) , (2) we get : $t_1 = \frac{2}{5}$, $t_2 = \frac{23}{5}$

by substitute in equation (3) :

$3 \times \frac{2}{5} - 2 \times \frac{23}{5} \neq -3$

i.e. These values don't satisfy equation (3)

$\therefore$ The two straight lines are skew.

6 (c)

7 (d)

Solution :

The matrix of coeffecients $A = \begin{pmatrix} 1 & 3 & 2 \\ 1 & 0 & 1 \\ 1 & 2 & 0 \end{pmatrix}$

$, |A| = \begin{vmatrix} 1 & 3 & 2 \\ 1 & 0 & 1 \\ 1 & 2 & 0 \end{vmatrix} = 5$

$, A^{adj} = \begin{pmatrix} -2 & 1 & 2 \\ 4 & -2 & 1 \\ 3 & 1 & -3 \end{pmatrix}^t = \begin{pmatrix} -2 & 4 & 3 \\ 1 & -2 & 1 \\ 2 & 1 & -3 \end{pmatrix}$

$A^{-1} = \frac{1}{5}\begin{pmatrix} -2 & 4 & 3 \\ 1 & -2 & 1 \\ 2 & 1 & -3 \end{pmatrix}$

8 (b)

Solution :

$T_{r+1} = {}^{12}C_r \left(\frac{3}{2x^2}\right)^r \left(\frac{2x}{3}\right)^{12-r}$

$= {}^{12}C_r \times 3^{2r-12} \times 2^{12-2r} \times x^{12-3r}$

To find the term whose x^{-3}

Put $12 - 3r = -3$ $\qquad \therefore r = 5$

$\therefore$ The term whose x^{-3} is $T_6 = {}^{12}C_5 \times \frac{4}{9} x^{-3} = 352\,x^{-3}$

$\because$ The middle term $= T_7 = {}^{12}C_6 \left(\frac{3}{2x^2}\right)^6 \left(\frac{2x}{3}\right)^6$

$= {}^{12}C_6\, x^{-6} = 924\,x^{-6}$

$\frac{T_7}{T_6} = \frac{12-6+1}{6} \times \frac{9}{4x^3} = \frac{7}{9}$

$\therefore x^3 = \frac{27}{8}$ $\qquad \therefore x = \frac{3}{2}$

9 (a)

Solution :

$\overrightarrow{CA} \cdot \overrightarrow{CB} = \|\overrightarrow{CA}\| \times \|\overrightarrow{CB}\| \cos C$

$b\,a \times \frac{a^2 + b^2 - c^2}{2ab} = \frac{1}{2}(a^2 + b^2 - c^2)$

10 (a)

11 (c)

Solution :

Centre of the sphere $m = (1, -2, 2)$

$\therefore 1 + 4 + 4 - k < 0$ $\qquad \therefore k < 9$

$\therefore$ k could by equal 5

12 (c)

Solution :

Direction vector of the straight line

$\vec{d} = (4, 3, -5) - (-2, 1, -8) = (6, 2, 3)$

the unit vector in direction $\vec{d} = \frac{(6, 2, 3)}{\sqrt{36 + 4 + 9}}$

$= \left(\frac{6}{7}, \frac{2}{7}, \frac{3}{7}\right)$

$\therefore$ The direction cosines of the straight line joining the two points $= \left(\frac{6}{7}, \frac{2}{7}, \frac{3}{7}\right)$

13 (c)

Solution :

$z_1 = \cos\left(\frac{-7\pi}{4}\right) + i \sin\left(\frac{-7\pi}{4}\right) = e^{\frac{-7\pi}{4}i} = e^{\frac{\pi}{4}i}$

$z_2 = -1 + i = \sqrt{2}\left(\cos\frac{3\pi}{4} + i \sin\frac{3\pi}{4}\right) = \sqrt{2}\,e^{\frac{3\pi}{4}i}$

$z = z_1^2\, z_2^6 = e^{\frac{\pi}{2}i} \times 8\,e^{\frac{9\pi}{2}i} = 8\,e^{5\pi i} = 8\,e^{\pi i}$

14 (b)

Solution :

$\begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix}$ (by doing $R_3 - R_2$ then $R_2 - R_1$)

$= \begin{vmatrix} 1 & a & bc \\ 0 & b-a & c(a-b) \\ 0 & c-b & a(b-c) \end{vmatrix}$

$= (b-a)(c-b)\begin{vmatrix} 1 & a & bc \\ 0 & 1 & -c \\ 0 & 1 & -a \end{vmatrix}$ (by doing $R_3 - R_2$)

$= (a-b)(b-c)\begin{vmatrix} 1 & a & bc \\ 0 & 1 & -c \\ 0 & 0 & c-a \end{vmatrix}$

$= (a-b)(b-c)(c-a)$

15 (c)

Solution :

$(3x - 2)^3 = 8$

$\because$ The cubic roots of 8 are $2, 2\omega, 2\omega^2$

$\therefore 3x - 2 = 2$, then $x = \frac{4}{3}$

or $3x - 2 = 2\omega$, then $x = \frac{2\omega + 2}{3} = \frac{-2\omega^2}{3}$

or $3x - 2 = 2\omega^2$, then $x = \frac{2\omega^2 + 2}{3} = \frac{-2\omega}{3}$

16 (c)

Solution :

$\overrightarrow{BA} = (2, 4, -1) - (-5, -3, 6) = (7, 7, -7)$

$\vec{d} = (1, 4, -9)$

$\therefore \overrightarrow{BA} \times \vec{d} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 7 & 7 & -7 \\ 1 & 4 & -9 \end{vmatrix} = -35\,\hat{i} + 56\,\hat{j} + 21\,\hat{k}$

length of the perpendicular $= \frac{\|\overrightarrow{AB} \times \vec{d}\|}{\|\vec{d}\|}$

$= \frac{\sqrt{(-35)^2 + (56)^2 + (21)^2}}{\sqrt{1^2 + 4^2 + (-9)^2}}$

$= 7$ length unit.

17 (c)

Solution :

$(x - 1)(x + 1) - 4 \times 2 = 0$

$x^2 - 1 - 8 = 0$ $\qquad \therefore x^2 = 9$ $\qquad \therefore x = \pm 3$

18 (b)

Solution :

$\left(\frac{1}{2}, \frac{3}{8}, \frac{9}{32}, \frac{27}{128}, \dots\right)$ is geometric sequence

its 1^{st} term is $a = \frac{1}{2}$

its common ratio is $r = \frac{3}{4}$

$\therefore \left(\frac{1}{2} + \frac{3}{8} + \frac{9}{32} + \frac{27}{128} + \dots\right) + \omega + \omega^2$

$= \frac{\frac{1}{2}}{1 - \frac{3}{4}} + (-1) = 2 - 1 = 1$

19 (b)

Solution :

$A \times A^{adj} = |A| \times I = 5\,I$

$\therefore |A \times A^{adj}| = |5\,I| = 5^2\,|I| = 25 \times 1 = 25$

20 (b)

Solution :

$B = (3, 5, 0)$ $\qquad \therefore \overrightarrow{OB} = (3, 5, 0)$

$B' = (3, 5, 4)$ $\qquad \therefore \overrightarrow{OB'} = (3, 5, 4)$

$\cos\theta = \frac{\overrightarrow{OB} \cdot \overrightarrow{OB'}}{\|\overrightarrow{OB}\|\,\|\overrightarrow{OB'}\|} = \frac{(3, 5, 0) \cdot (3, 5, 4)}{\sqrt{9 + 25} \times \sqrt{9 + 25 + 16}}$

$= \frac{34}{10\sqrt{17}} = \frac{\sqrt{17}}{5}$

$\therefore \theta \simeq 34^\circ\ 27'$

21 (d)

Solution :

Direction vector of given straight line $= (-3, 2, 1)$

$\therefore$ Direction vector of the required straight line is $(-3, 2, 1)$

$\therefore$ The equation of the straight line is :

$\frac{x}{-3} = \frac{y-7}{2} = \frac{z+7}{1}$

22 (b)

Solution :

$\frac{(2n)!}{3\,(2n-3)!} \div \frac{n!}{2\,(n-2)!} \le 11$

$\therefore \frac{2n(2n-1)(2n-2)\,(2n-3)!}{3\,2!\,(2n-3)!} \times \frac{2!\,(n-2)!}{n(n-1)\,(n-2)!} \le 11$

$\therefore \frac{4(2n-1)(n-1)}{3(n-1)} \le 11$ $\qquad \therefore 8n - 4 \le 33$

$\therefore 8n \le 37$ $\qquad \therefore n \le \frac{37}{8}$

$\therefore n = 4$

23 (a)

Solution :

$T_{r+1} = {}^9C_r \times \left(\frac{-1}{3x}\right)^r \times \left(\frac{2}{3}x^2\right)^{9-r}$

$= {}^9C_r \times \left(\frac{-1}{3}\right)^r \times x^{-r} \times \left(\frac{3}{2}\right)^{9-r} \times x^{18-2r}$

$= {}^9C_r \times \left(\frac{-1}{3}\right)^r \times \left(\frac{3}{2}\right)^{9-r} \times x^{18-3r}$

$\therefore 18 - 3r = 0$ $\qquad \therefore r = 6$

$\therefore T_7$ is the term free of x

$, T_7 = {}^9C_6 \times \left(\frac{-1}{3}\right)^6 \times \left(\frac{3}{2}\right)^3 = \frac{7}{18}$

24 (a)

Solution :

$\begin{vmatrix} a & a & a \\ a & b & c \\ c & c & c \end{vmatrix} = a \times c \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix}$

$\because (R_1 = R_3)$ $\qquad \therefore \Delta = a \times c \times \text{zero} = \text{zero}$

25 (b)

Solution :

let $x + y\,i = \sqrt{5 + 12\,i}$ where $x, y \in R$

$\therefore (x + y\,i)^2 = 5 + 12\,i$

$\therefore x^2 - y^2 + 2xy\,i = 5 + 12\,i$

$\therefore x^2 - y^2 = 5 \quad (1)$, $2xy = 12$

$xy = 6 \qquad (2)$

From (1) , (2) :

$\therefore x^2 - \frac{36}{x^2} = 5 \qquad \therefore x^4 - 5x^2 - 36 = 0$

$\therefore (x^2+4)(x^2-9) = 0$

$\therefore x^2 + 4 = 0$, then $x = \pm 2i$ (refused)

or $x^2 - 9 = 0$

$\therefore x = \pm 3 \; , \; y = \pm \frac{6}{3} = \pm 2$

$\therefore x + yi = \pm(3+2i) \qquad \therefore \sqrt{5+12i} = \pm(3+2i)$

Note that we can square the answers to chech the correct answer without using previous steps

i.e. $(\pm(3+2i))^2 = 9 - 4 + 12i = 5 + 12i$

$\therefore \sqrt{5+12i} = \pm(3+2i)$

Exam 8

1 (b)

Solution :

A (2 , −1 , 1) , B (1 , −3 , −5) , C (3 , −4 , −4)

equation of the plane is

$$\begin{vmatrix} x-2 & y+1 & z-1 \\ -1 & -2 & -6 \\ 1 & -3 & -5 \end{vmatrix} = 0$$

$\therefore -8(x-2) - 11(y+1) + 5(z-1) = 0$

$-8x - 11y + 5z = 0$

2 (a)

Solution :

$\because z_1 = -1 + \sqrt{3}\,i \qquad \therefore r = \sqrt{(-1)^2 + (\sqrt{3})^2} = 2$

$\because x < 0 \; , \; y > 0$

$\therefore \theta = \pi + \tan^{-1}\left(\frac{\sqrt{3}}{-1}\right) = \pi - \frac{\pi}{3} = \frac{2}{3}\pi$

$\therefore z_1 = 2\left(\cos\frac{2}{3}\pi + i\sin\frac{2}{3}\pi\right) = 2e^{\frac{2}{3}\pi i}$

3 (a)

Solution :

Let the three terms are T_r , T_{r+1} , T_{r+2} respectively

$\therefore \frac{\text{coeff. of } T_{r+1}}{\text{coeff. of } T_r} = \frac{n-r+1}{r} \times \frac{1}{1} = \frac{21}{35} = \frac{3}{5}$

$\therefore 5n - 5r + 5 = 3r$

$\therefore 5n - 8r + 5 = 0$ (1)

, $\frac{\text{coeff. of } T_{r+2}}{\text{coeff. of } T_{r+1}} = \frac{n-(r+1)+1}{r+1} \times \frac{1}{1} = \frac{7}{21} = \frac{1}{3}$

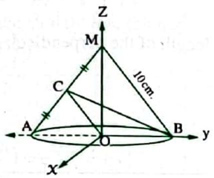
$\therefore 3n - 3r = r + 1$

$3n - 4r - 1 = 0$ (2)

From (1) , (2) : $\therefore n = 7 \; , \; r = 5$

4 (a)

Solution :



$2\pi r = 12\pi$

$\therefore r = 6$ cm.

$\therefore OB = 6$ cm.

, $MO = \sqrt{10^2 - 6^2} = 8$ cm.

From the figure

B = (0 , 6 , 0) , C (0 , −3 , 4)

, O (0 , 0 , 0)

$\therefore \overrightarrow{BC} = (0, -9, 4) \; , \; \overrightarrow{CO} = (0, 3, -4)$

$\overrightarrow{BC} \cdot \overrightarrow{CO} = (0, -9, 4) \cdot (0, 3, -4)$

$= 0 - 27 - 16 = -43$

5 (b)

Solution :

$\because$ The point (1 , 2 , 3) lies on each of the two straight lines

$\because \overrightarrow{d_1} = (2, 4, 7) \; , \; \overrightarrow{d_2} = (4, 5, 7)$

$\therefore \overrightarrow{d_1} \cdot \overrightarrow{d_2} \neq$ zero

$\therefore$ The two straight lines are not perpendicular

$\because \frac{2}{4} \neq \frac{4}{5} \neq \frac{7}{7}$

$\therefore$ The two straight lines are not parallel

$\therefore$ The two straight lines are intersecting

6 (c)

Solution :

$\because \cos^2 45 + \cos^2 60 + \cos^2 120 = \frac{1}{2} + \frac{1}{4} + \frac{1}{4} = 1$

$\therefore$ 45° , 60° , 120° are direction angles of the vector

7 (d)

Solution :

Put $x = 1 \qquad \therefore \begin{vmatrix} 0 & 1 & 1 \\ 1 & 1 & 2 \\ -1 & 1 & 1+k \end{vmatrix} =$ zero

$\therefore -(1+k+2) + 1(1+1) = 0$

$\therefore -k - 3 + 2 = 0 \qquad \therefore k = -1$

8 (c)

Solution :

$T_5 = {}^{10}C_4 (2x)^4 = {}^{10}C_4\, 2^4 x^4 = 16 \times {}^{10}C_4\, x^4$

$\therefore$ Coefficient $T_5 = 16 \times {}^{10}C_4$

9 (c)

Solution :

$\because RK(A) = 2 \qquad \therefore \begin{vmatrix} 1 & -2 & 3 \\ k & 0 & 1 \\ 3 & 2 & -1 \end{vmatrix} =$ zero

$\therefore 1(0-2) + 2(-k-3) + 3(2k-0) =$ zero

$-2 - 2k - 6 + 6k = 0$

$\therefore k = 2$

10 (c)

Solution :

$A - A^2 = I \qquad \therefore A(I - A) = I$

$\therefore A^{-1} = I - A$

11 (c)

Solution :

$\therefore {}^{n-1}C_6 + {}^{n-1}C_7 > {}^{n}C_6$

$\therefore {}^{n}C_7 > {}^{n}C_6 \qquad \therefore \frac{{}^{n}C_7}{{}^{n}C_6} > 1$

$\therefore \frac{n-7+1}{7} > 1 \qquad \therefore n > 13$

12 (b)

Solution :

$$\overrightarrow{B} \times \overrightarrow{C} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 2 \\ 1 & 2 & 1 \end{vmatrix} = -\hat{i} + \hat{k}$$

$\overrightarrow{A} = \frac{\overrightarrow{B} \times \overrightarrow{C}}{\|\overrightarrow{B} \times \overrightarrow{C}\|} \times \|\overrightarrow{A}\| = \frac{(-1, 0, 1)}{\sqrt{2}} \times 4\sqrt{2}$

$= (-4, 0, 4)$

13 (d)

Solution :

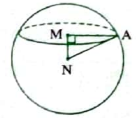
$(i^{21})^3 = i^{63} = i^3 = -i$

the trigonometric form $= \left(\cos\frac{-\pi}{2} + i\sin\frac{-\pi}{2}\right)$

$= \cos\frac{\pi}{2} - i\sin\frac{\pi}{2}$

14 (c)

Solution :



Centre of the sphere is N (0 , 1 , 2)

and length of its radius

$AN = \sqrt{1^2 + 2^2 - (-11)}$

$= 4$ length unit.

When the plane intersect the sphere , then the section is a circle , the straight line which passes the centres of circle and sphere is perpendicular to the plane of circle.

$\therefore$ MN = length of perpendicular line segment from N to the plane

$= \frac{|0 + 2(1) + 2(2) - 15|}{\sqrt{1+4+4}} = 3$ length unit.

Radius length of the circle $= \sqrt{4^2 - 3^2} = \sqrt{7}$ length unit.

15 (c)

Solution :

$\because \begin{vmatrix} 2 & y \\ 1 & x \end{vmatrix} = 1 \qquad \therefore 2x - y = 1$

$, \begin{vmatrix} 3 & z \\ 2 & y \end{vmatrix} = 1 \qquad \therefore 3y - 2z = 1$

$, \begin{vmatrix} 3 & x \\ 1 & z \end{vmatrix} = 2 \qquad \therefore -x + 3z = 2$

The matrix equation is

$$\begin{pmatrix} 2 & -1 & 0 \\ 0 & 3 & -2 \\ -1 & 0 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$$

$$\because |A| = \begin{vmatrix} 2 & -1 & 0 \\ 0 & 3 & -2 \\ -1 & 0 & 3 \end{vmatrix} = 16 \neq 0$$

$$\because \text{Adj}(A) = \begin{pmatrix} 9 & 2 & 3 \\ 3 & 6 & 1 \\ 2 & 4 & 6 \end{pmatrix}^T = \begin{pmatrix} 9 & 3 & 2 \\ 2 & 6 & 4 \\ 3 & 1 & 6 \end{pmatrix}$$

$$A^{-1} = \frac{1}{16} \begin{pmatrix} 9 & 3 & 2 \\ 2 & 6 & 4 \\ 3 & 1 & 6 \end{pmatrix}$$

$$\therefore \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{16} \begin{pmatrix} 9 & 3 & 2 \\ 2 & 6 & 4 \\ 3 & 1 & 6 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$\therefore x = 1 \quad , \quad y = 1 \quad , \quad z = 1$

$\therefore x + y + z = 3$

---

16 (c)

Solution :

$A(4, 0, 0), B(0, 2, 0), C(0, 0, 2)$

$\therefore \overrightarrow{AB} = (-4, 2, 0), \overrightarrow{AC} = (-4, 0, 2)$

$$\therefore \overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -4 & 2 & 0 \\ -4 & 0 & 2 \end{vmatrix} = 4\hat{i} + 8\hat{j} + 8\hat{k}$$

$$\text{Area of } \Delta ABC = \frac{1}{2} \| \overrightarrow{AB} \times \overrightarrow{AC} \| = \frac{1}{2}\sqrt{4^2 + 8^2 + 8^2}$$

$= 6$ square unit.

---

17 (d)

Solution :

$z_1 = 8\left(\cos(180° - \theta) + i \sin(180° - \theta)\right)$

$, z_2 = 4\left(\cos -(90° + \theta) + i \sin -(90° + \theta)\right)$

$\therefore \frac{z_1}{z_2} = 2(\cos 270° + i \sin 270°) = -2i$

---

18 (c)

Solution :

$\because$ Point of division is $(0, y, z)$

$\frac{2\ell_2 + 3\ell_1}{\ell_1 + \ell_2} = 0 \qquad \therefore 2\ell_2 + 3\ell_1 = 0$

$\therefore 2\ell_2 = -3\ell_1 \qquad \therefore \frac{\ell_1}{\ell_2} = \frac{-2}{3}$

---

19 (a)

Solution :

The term which has the greatest coefficient in the expansion $(x + y)^n$ is the middle term

$\therefore F_7$ is the middle term in the expansion

$\therefore \frac{n+2}{2} = 7 \qquad \therefore n = 12$

---

20 (b)

Solution :

The centre of the sphere $M(2, -4, -3)$

, its radius $r = \sqrt{4} = 2$ length unit.

$\therefore$ The distance of the centre to the plane

$yz = |2| = 2 = r$

$\therefore$ The sphere touches the plane yz

---

21 (a)

Solution :

$$\begin{vmatrix} 1 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix} = \begin{vmatrix} \omega^3 & \omega & \omega^2 \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix}$$

«ω is a common factor»

$$= \omega \begin{vmatrix} \omega^2 & 1 & \omega \\ \omega & \omega^2 & 1 \\ \omega^2 & 1 & \omega \end{vmatrix} \quad «\because R_1 = R_3»$$

$\therefore \omega \times \text{zero} = \text{zero}$

---

22 (c)

Solution :

$T_2 = {}^nC_1 (a x)^1 = n a x \qquad \therefore n a = 8 \qquad (1)$

$T_3 = {}^nC_2 (a x)^2 = \frac{n(n-1)}{2} \times a^2 x^2$

$\therefore \frac{n(n-1)}{2} \times a^2 = 24 \qquad \because n a \times (n a - a) = 48 \ (2)$

by substituting from (1) in (2)

$\therefore 8(8 - a) = 48 \qquad \therefore 8 - a = 6 \qquad \therefore a = 2$

$, n = \frac{8}{2} = 4$

$\therefore \frac{a-n}{a+n} = \frac{2-4}{2+4} = \frac{-1}{3}$

---

23 (d)

---

24 (c)

Solution :

$\overrightarrow{d_1} = (2, 0, -2) \quad , \quad \overrightarrow{d_2} = (1, 2, -2)$

$$\cos\theta = \frac{|(2, 0, -2) \cdot (1, 2, -2)|}{\sqrt{4+0+4} \times \sqrt{1+4+4}} = \frac{1}{\sqrt{2}}$$

$\therefore \theta = 45°$

---

25 (b)

Solution :

$a + b\omega = (1 + \omega)^7 = (-\omega^2)^7 = -\omega^{14} = -\omega^2 = 1 + \omega$

$\therefore a = 1 \quad , \quad b = 1$

$\therefore (a, b) = (1, 1)$

Exam 9

1 (d)

Solution :

$$T_{r+1} = {}^{12}C_r \times \left(\frac{-1}{2x^2}\right)^r \times (2x)^{12-r}$$

$$= {}^{12}C_r \times (-1)^r \times 2^{12-2r} \times x^{12-3r}$$

Put : $12 - 3r = 0 \qquad \therefore r = 4$

$\therefore T_5$ is the term free of x

$\therefore T_5 = {}^{12}C_4 (-1)^4 \times 2^4 = {}^{12}C_4 \times 2^4$

---

2 (a)

Solution :

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ a+1 & b+1 & c+1 \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ a & b & c \end{vmatrix} + \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 1 & 1 \end{vmatrix}$$

$= k + 0 = k$

---

3 (c)

Solution :

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 12 \\ 7 \end{pmatrix}$$

$\therefore y = 7$

---

4 (d)

---

5 (a)

Solution :

$|z| = |\omega|^x = 1^x = 1$

---

6 (b)

Solution :

The distance of the A to x-axis $= \sqrt{9 + 16}$

$= 5$ length unit

---

7 (c)

Solution :

Put $x = 1$

$\therefore$ Sum of coefficients of terms to the expansion $(a^2 - 2a + 1)^{51}$

$\therefore (a^2 - za + 1)^{51} = 0 \qquad \therefore a^2 - 2a + 1 = 0$

$\therefore (a - 1)^2 = 0$

i.e. $a = 1$

---

8 (b)

Solution :

$\because \frac{2}{6} = \frac{-1}{-3} = \frac{3}{9} \neq \frac{-4}{13}$

$\therefore$ The two planes are parallel and not coincident

$\therefore$ Find a point belongs to the 1^{st} plane

Let $x = 0 \quad , \quad y = 0 \qquad \therefore z = \frac{4}{3}$

$\therefore \left(0, 0, \frac{4}{3}\right) \in 1^{st}$ plane

$\therefore$ Distant between two planes

$$= \frac{\left|6(0) - 3(0) + 9\left(\frac{4}{3}\right) + 13\right|}{\sqrt{36 + 9 + 81}} = \frac{25}{\sqrt{126}} \text{ length unit.}$$

---

9 (b)

Solution :

$\overrightarrow{d_1} = \left(1, \sqrt{2}, -1\right) \quad , \quad \overrightarrow{d_2} = (-1, 0, 1)$

$$\cos\theta = \frac{\left|\left(1, \sqrt{2}, -1\right) \cdot (-1, 0, 1)\right|}{\sqrt{1+2+1} \times \sqrt{1+1+\text{zero}}} = \frac{|-2|}{2\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$\therefore \theta = 45°$

---

10 (c)

Solution :

$\because \overline{AC} \perp \overline{BD} \qquad \therefore \overrightarrow{AC} \cdot \overrightarrow{BD} = \text{zero}$

$\therefore \left(\vec{C} - \vec{A}\right) \cdot \left(\vec{D} - \vec{B}\right) = 0$

---

11 (b)

Solution :

$\because (-3, 2, 6)$ is direction ratios of the perpendicular straight line to the plane "P"

$\therefore \vec{n} = (-3, 2, 6) \perp$ plane P

$\therefore$ The equation of the plane P in form

$-3x + 2y + 6z + d = 0$

$\because$ The length of perpendicular segment from the origin to the plane P = 7 units

$$= \frac{|-3(0) + 2(0) + 6(0) + d|}{\sqrt{9 + 4 + 36}} = 7$$

$\therefore |d| = 49 \qquad \therefore d = \pm 49$

$\therefore$ The equation of the plane is

$-3x + 2y + 6z \pm 49 = 0$

12 (c)

Solution :

$$z=\left(1+\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)^6=\left(\frac{3}{2}+\frac{\sqrt{3}}{2}i\right)^6$$
$$=\left(\sqrt{3}\right)^6\left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)^6$$
$$=27\,(\cos 30+i\sin 30)^6$$
$$=27\,(\cos 180+i\sin 180)$$

$\therefore r=27$, $\theta=\pi$

, then the number in exponential form is $27\,e^{\pi i}$

13 (b)

Solution :

The two middle terms are T_6 , T_7

$\therefore T_6+T_7=0$, $\frac{T_7}{T_6}=-1$

$\therefore \frac{11-6+1}{6}\times\frac{-4}{x}\times\frac{2}{x^3}=-1$

$\therefore \frac{-8}{x^4}=-1$ $\quad\therefore x^4=8$ $\quad\therefore x=\pm\sqrt[4]{8}$

14 (d)

Solution :

$\because \overrightarrow{AB}=\vec{B}-\vec{A}=(2,0,0)-(0,1,2)$
$=(2,-1,-2)$

$\therefore \|\overrightarrow{AB}\|=3$

$,\ \vec{F}=\|\vec{F}\|\times\frac{\overrightarrow{AB}}{\|\overrightarrow{AB}\|}=21\times\frac{(2,-1,-2)}{3}$
$=(14,-7,-14)$

15 (b)

Solution :

$\overrightarrow{v_A}=\frac{(-2,1,2)}{\sqrt{4+1+4}}=\left(\frac{-2}{3},\frac{1}{3},\frac{2}{3}\right)$

$\therefore$ The direction cosines of $\vec{A}=\left(\frac{-2}{3},\frac{1}{3},\frac{2}{3}\right)$

16 (d)

Solution :

$$\Delta=\begin{vmatrix}1&\omega^n&\omega^{2n}\\ \omega^n&\omega^{2n}&1\\ \omega^{2n}&1&\omega^n\end{vmatrix}\quad \text{"Put } 1=\omega^{3n}\text{"}$$

$$=\begin{vmatrix}\omega^{3n}&\omega^n&\omega^{2n}\\ \omega^n&\omega^{2n}&\omega^{3n}\\ \omega^{2n}&\omega^{3n}&\omega^n\end{vmatrix}$$

"take ω^n a common factor from R_1"

$$=\omega^n\begin{vmatrix}\omega^{2n}&1&\omega^n\\ \omega^n&\omega^{2n}&1\\ \omega^{2n}&1&\omega^n\end{vmatrix}\quad \text{"}\because R_1=R_3\text{"}$$

$=\omega^n\times\text{zero}=\text{zero}$

17 (a)

Solution :

Let $A=(x_1,y_1,z_1)$

$\therefore \overrightarrow{BA}=\vec{A}-\vec{B}=(x_1-2\ ,\ y_1-1\ ,\ z_1-6)$

$,\because \overrightarrow{BA}\perp$ the plane

$\therefore \overrightarrow{BA}$ parallels the normal direction vector of the plane.

$\therefore \overrightarrow{BA}$ parallels the vector $(1,1,-2)$

$\therefore \frac{x_1-2}{1}=\frac{y_1-1}{1}=\frac{z_1-6}{-2}=t$

$\therefore x_1=t+2$, $y_1=t+1$, $z_1=-2t+6$

$,\because (x_1,y_1,z_1)\in$ the plane

$\therefore t+2+t+1-2(-2t+6)=3$

$\therefore 6t-9=3$ $\qquad\therefore t=2$

$\therefore x_1=4$, $y_1=3$, $z_1=2$

$\therefore$ The point A $(4,3,2)$

18 (b)

Solution :

$\because {}^nC_{r+2}={}^nC_{r+4}$ $\qquad\therefore r+2=r+4$ (refused)

or $n=2r+6$ $\qquad$ (1)

$,\because \frac{{}^nC_{r+2}}{{}^nC_r}=\frac{14}{3}$ $\qquad\therefore \frac{{}^nC_{r+2}}{{}^nC_{r+1}}\times\frac{{}^nC_{r+1}}{{}^nC_r}=\frac{14}{3}$

$\therefore \frac{n-r-2+1}{r+2}\times\frac{n-r-1+1}{r+1}=\frac{14}{3}$

From (1) $\therefore \frac{r+5}{r+2}\times\frac{r+6}{r+1}=\frac{14}{3}$

$\therefore 14(r^2+3r+2)=3(r^2+11r+30)$

$\therefore 11r^2+9r-62=0$ $\qquad\therefore (11r+31)(r-2)=0$

$\therefore r=2$

From (1) $\therefore n=10$ $\qquad\therefore {}^nP_r={}^{10}P_2=90$

19 (d)

Solution :

The equation of the sphere is

$(x+2)^2+(y-1)^2+(z+4)^2=625$

$x^2+y^2+z^2+4x-2y+8z+21-625=0$

$x^2+y^2+z^2+4x-2y+8z-604=0$

20 (c)

Solution :

The direction vector of the straight line

is $\vec{d_1}=(-1,1,1)$

The normal direction vector of the plane $\hat{n}=(3,2,-1)$

$\because \cos\theta=\frac{|(-1,1,1).(3,2,-1)|}{\sqrt{1+1+1}\times\sqrt{9+4+1}}$

$=\frac{|-3+2-1|}{\sqrt{3}\times\sqrt{14}}=\frac{2}{\sqrt{42}}$

$,\because$ measure of the angle between the straight line and the plane $\propto=90°-\theta$

$\therefore \sin\propto=\cos\theta=\frac{2}{\sqrt{42}}$ $\qquad\therefore x=\sin^{-1}\left(\frac{2}{\sqrt{42}}\right)$

21 (b)

Solution :

$$A^*=\left(\begin{array}{cc|c}2&-3&5\\6&-9&15\end{array}\right)$$

$\because \begin{vmatrix}2&-3\\6&-9\end{vmatrix}=0$, $\begin{vmatrix}-3&5\\-9&15\end{vmatrix}=0$

$\therefore \begin{vmatrix}2&5\\6&15\end{vmatrix}=0$ $\qquad\because A^*$ is non zero matrix

$\therefore RK(A^*)=1$

22 (c)

Solution :

Let amplitude of $z_1=\theta$

$\therefore$ Amplitude of $z_1z_2=90°+\theta$

$\therefore z_1=r(\cos\theta+i\sin\theta)$

$\therefore z_1z_2=r\left(\cos(90°+\theta)+i\sin(90°+\theta)\right)$

$\because z_2=\frac{z_1z_2}{z_1}=\frac{r\left(\cos(90°+\theta)+i\sin(90°+\theta)\right)}{r(\cos\theta+i\sin\theta)}$

$=\cos 90°+i\sin 90°=i$

23 (d)

Solution :

$r^2=r+2$ $\qquad\therefore r^2-r-2=0$

$\therefore (r-2)(r+1)=0$ $\qquad\therefore r=2$ or $r=-1$

or $r^2+r+2=14$ $\qquad\therefore r^2+r-12=0$

$(r+4)(r-3)=0$ $\qquad\therefore r=-4$ (refused) or $r=3$

$\therefore r=2$ or 3 or -1

24 (a)

Solution :

$\left(\frac{-1+\sqrt{-3}}{2}\right)^5+\left(\frac{-1-\sqrt{-3}}{2}\right)^8$

$=\left(\frac{-1}{2}+\frac{\sqrt{3}}{2}i\right)^5+\left(\frac{-1}{2}-\frac{\sqrt{3}}{2}i\right)^8$

$=\omega^5+(\omega^2)^8=\omega^2+\omega=-1$

25 (d)

Solution :

$$A=\begin{pmatrix}3&1&-1\\5&2&-3\\15&6&-9\end{pmatrix}$$

$$|A|=\begin{vmatrix}3&1&-1\\5&2&-3\\15&6&-9\end{vmatrix}=\text{zero}$$

$\because \begin{vmatrix}3&1\\5&2\end{vmatrix}=6-5=1\neq 0$

$\therefore RK(A)=2$

$$A^*=\left(\begin{array}{ccc|c}3&1&-1&0\\5&2&-3&2\\15&6&-9&5\end{array}\right)$$

$\because \begin{vmatrix}1&-1&0\\2&-3&2\\6&-9&5\end{vmatrix}=1(-15+18)+1(10-12)=1\neq 0$

$\therefore RK(A^*)=3$ $\qquad\therefore RK(A)\neq RK(A^*)$

$\therefore$ The system has no any solution.

Exam 10

1 (b)

Solution :

$$\begin{vmatrix} 1 & a & b+c \\ 1 & b & a+c \\ 1 & c & a+b \end{vmatrix} = \begin{vmatrix} 1 & a+b+c & b+c \\ 1 & a+b+c & a+c \\ 1 & a+b+c & a+b \end{vmatrix}$$

$$= (a+b+c)\begin{vmatrix} 1 & 1 & b+c \\ 1 & 1 & a+c \\ 1 & 1 & a+b \end{vmatrix}$$

$$= (a+b+c) \times \text{zero} = \text{zero}$$

2 (c)

Solution :

$z_1 + z_2 = -1 - \sqrt{3}\, i$

$x < 0$, $y < 0$

$\therefore \theta = -\pi + \tan^{-1}\left(\frac{-\sqrt{3}}{-1}\right) = -\pi + \frac{\pi}{3} = \frac{-2}{3}\pi$

$\therefore$ Amplitude of $(z_1 + z_2) = \frac{-2}{3}\pi$

3 (b)

Solution :

$a + b\,\omega = (1+\omega)^7 = (-\omega^2)^7 = -\omega^{14} = -\omega^2 = 1 + \omega$

$\therefore a = 1$, $b = 1$ $\qquad \therefore (a , b) = (1 , 1)$

4 (b)

5 (a)

Solution :

$m_1 = (2 , -4 , 2)$, $r_1 = 1$

, $m_2 = (-4 , 4 , 2)$, $r_2 = 2$

$\therefore m_1m_2 = \sqrt{6^2 + (-8)^2 + 0^2} = 10$ units.

, $\because r_1 + r_2 < m_1m_2$

$\therefore$ The two spheres are distant.

6 (b)

Solution :

$\overrightarrow{A} \times (\overrightarrow{A} - \overrightarrow{B}) = (1 , 1 , 1) \times (-1 , 2 , 2)$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ -1 & 2 & 2 \end{vmatrix} = -3\hat{j} + 3\hat{k}$$

7 (c)

Solution :

$$\because A = \begin{pmatrix} k & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & k \end{pmatrix}$$

$$\therefore \Delta = |A| = \begin{vmatrix} k & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & k \end{vmatrix} = k^3$$

$\because A \,.\, A^{adj} = \Delta\, I$

$\therefore |A|\,|A^{adj}| = |AA^{adj}| = |k^3\, I| = k^9$

8 (b)

Solution :

Put $x = 2$ $\qquad \therefore$ The value of the determinant $= 0$

$$\therefore \begin{vmatrix} 1 & 5 & 2 \\ -3 & 7 & -6 \\ 5 & 2 & 2+k \end{vmatrix} = 0$$

$\therefore (14 + 7k + 12) - 5(-6 - 3k + 30)$
$\qquad + 2(-6 - 35) = 0$

$\therefore k = 8$

9 (b)

Solution :

$(n+2)(n+1) + 3n = 74$ $\qquad \therefore n^2 + 6n - 72 = 0$

$\therefore (n+12)(n-6) = 0$ $\qquad \therefore n = 6$

, $\because \lfloor r = n$ $\qquad \therefore \lfloor r = 6$

$\therefore r = 3$ $\qquad \therefore {}^{n}C_r = {}^{6}C_3 = 20$

10 (c)

Solution :

The coordinates of the point A (midpoint of $\overline{BC}$)

$= \left(\frac{2+6}{2} , \frac{3+7}{2} , \frac{4+8}{2}\right) = (4 , 5 , 6)$

, by substituting in the given equations its found that $(4 , 5 , 6)$ satisfies equation of the plane

$x + y + 7 - 15 = 0$

11 (a)

Solution :

$\overrightarrow{d_1} = (0 , 1 , 1)$, $\overrightarrow{d_2} = (1 , 0 , 1)$

and let $\propto$ is the measure of the acute angle between the two straight lines.

$$\therefore \cos \propto = \frac{|(0 , 1 , 1) \,.\, (1 , 0 , 1)|}{\sqrt{0+1+1}\sqrt{1+0+1}} = \frac{1}{2}$$

$\therefore \propto = 60°$

$\therefore \theta = 180° - 60° = 120°$

12 (a)

Solution :

$T_{r+1} = {}^{18}C_r\, (x^{-1})^r\, (x^2)^{18-r} = {}^{18}C_r\, x^{36-3r}$

Put $36 - 3r = 0$ $\qquad \therefore r = 12$

$\therefore T_{13}$ is the term free of x

, $\because$ the middle term is T_{10} $\qquad \therefore \frac{T_{13}}{\text{coeff.}\, T_{10}} = \frac{{}^{18}C_{12}}{{}^{18}C_9} = \frac{21}{55}$

13 (a)

Solution :

The direction vector of the intersecting line

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -1 \\ 1 & -1 & 5 \end{vmatrix} = -6\hat{i} + 4\hat{j} - 2\hat{k}$$

i.e. $(3 , -2 , 1)$ is also the direction vector of the intersecting line.

, $\because (3 , -2 , 1)$ is the direction vector of the given straight line.

$\therefore$ The intersecting line // the given straight line.

14 (d)

Solution :

The matrix equation is

$$\begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & -3 \\ 1 & 1 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5 \\ 1 \\ 7 \end{pmatrix}$$

$$, \because |A| = \begin{vmatrix} 1 & 0 & 2 \\ 0 & 1 & -3 \\ 1 & 1 & 0 \end{vmatrix} = 1$$

$$, \because \text{Adj}\,(A) = \begin{pmatrix} 3 & -3 & -1 \\ 2 & -2 & -1 \\ -2 & 3 & 1 \end{pmatrix}^t = \begin{pmatrix} 3 & 2 & -2 \\ -3 & -2 & 3 \\ -1 & -1 & 1 \end{pmatrix}$$

$$, A^{-1} = \frac{1}{1}\begin{pmatrix} 3 & 2 & -2 \\ -3 & -2 & 3 \\ -1 & -1 & 1 \end{pmatrix}$$

$$\therefore \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 & 2 & -2 \\ -3 & -2 & 3 \\ -1 & -1 & 1 \end{pmatrix}\begin{pmatrix} 5 \\ 1 \\ 7 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix}$$

$\therefore x = 3$, $y = 4$, $z = 1$ $\qquad \therefore$ S.S. $= \{(3 , 4 , 1)\}$

15 (c)

Solution :

The work $= 120 \times 3 \cos 150° = -180\sqrt{3}$ Joule.

16 (c)

Solution :

Number of ways $= {}^{4+3-1}C_3 = {}^{6}C_3$

17 (b)

Solution :

The equation of the plane is

$(0 , 1 , -2) \,.\, (x , y , z) = (0 , 1 , -2) \,.\, (0 , 0 , 0)$

$\therefore y - 2z = 0$ $\qquad \therefore y = 2z$

18 (a)

Solution :

$y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}$ $\qquad \therefore 0 = \frac{3m_1 + m_2}{m_1 + m_2}$

$\therefore 3m_1 + m_2 = 0$ $\qquad , \therefore 3m_1 = -m_2$

$\therefore \frac{m_1}{m_2} = \frac{-1}{3}$

19 (d)

Solution :

$$\begin{vmatrix} -1 & 2 & 5 \\ 2 & -4 & a-4 \\ 1 & -2 & a+1 \end{vmatrix} = -2\begin{vmatrix} -1 & -1 & 5 \\ 2 & 2 & a-4 \\ 1 & 1 & a+1 \end{vmatrix}$$

$$= -2 \times \text{zero} = \text{zero}$$

$\forall$ values of a

, at $a = 6$

$\therefore \begin{vmatrix} -4 & 2 \\ -2 & 7 \end{vmatrix} \neq 0$ $\qquad \therefore$ RK (A) $= 2$

, at $a = -1$ $\qquad \therefore \begin{vmatrix} -4 & -5 \\ -2 & 0 \end{vmatrix} \neq 0$

$\therefore$ RK (A) $= 2$

, at $a = 2$ $\qquad \therefore \begin{vmatrix} -4 & -2 \\ -2 & 3 \end{vmatrix} \neq 0$

$\therefore$ RK (A) $= 2$

, at a = – 6 $\therefore \begin{vmatrix} -4 & -10 \\ -2 & -5 \end{vmatrix} = 0$

, $\because \begin{vmatrix} -1 & 5 \\ 2 & -10 \end{vmatrix} = 0$

and there is not exist a determinent in 2^{nd} degree its value = 0 $\therefore$ RK (A) = 1

20 (d)

Solution :

$(1+x)^n = {}^nC_0 + {}^nC_1\, x + {}^nC_2\, x^2 + \cdots + {}^nC_n\, x^n$

Put $x = 2$

$\therefore {}^nC_0 + 2\,{}^nC_1 + 2^2\,{}^nC_2 + \cdots + 2^n\,{}^nC_n = (1+2)^n = 3^n$

21 (d)

Solution :

$\hat{i}\cdot(\hat{j}\times\hat{k}) + \hat{j}\cdot(\hat{k}\times\hat{j}) + \hat{k}\cdot(\hat{i}\times\hat{j}) = \hat{i}\cdot\hat{i} + \hat{j}\cdot\hat{j} + \hat{k}\cdot\hat{k}$
$= 1 + 1 + 1 = 3$

22 (b)

Solution :

$z = 2(\omega + i)(\omega^2 + i) = 2(\omega^3 + \omega i + i\omega^2 + i^2)$
$= 2\left(1 + (-i) + (-1)\right) = -2i$

$\therefore |z| = 2$, amplitude of (z) $= \frac{-\pi}{2}$

$\therefore z = 2e^{\frac{-\pi}{2} i}$

23 (b)

Solution :

$z = 1 + \cos 40° + i \sin 40°$
$= 1 + 2\cos^2 20 - 1 + i(2\sin 20° \cos 20°)$
$= 2\cos 20° (\cos 20° + i \sin 20°)$

$\therefore$ Amplitude of (z) $= \frac{\pi}{9}$

24 (b)

Solution :

$\because \frac{T_5}{T_4} = 12$ $\therefore \frac{9-4+1}{4} \times (-x^2) \times x = 12$

$\therefore \frac{-3}{2} x^3 = 12$ $\therefore x^3 = -8$ $\therefore x = -2$

25 (b)

Solution :

$\because {}^7C_r > 1$ $\therefore r < 7$ $\because {}^7C_5 > 1$

$\therefore r > 5$ $\therefore 5 < r < 7$ $\therefore r = 6$

$\therefore \lfloor 6-r = \lfloor 6-6 = \lfloor 0 = 1$

Exam 11

1 (b)

2 (c)

Solution :

$z = \cos 30° - i \sin 30° = \cos(-30°) + i \sin(-30°)$

$\therefore$ Amplitude of $z = \frac{-\pi}{6}$

3 (d)

4 (b)

Solution :

Equation of plane is : $2x - 2y + z + 10 = 0$

$\therefore$ The centre of the sphere (1 , 2 , 1)

The radius of the sphere $= \sqrt{k}$ length unit.

, $\because$ the sphere touch the plane.

$\therefore$ length of perpendicular line segment from the centre to the plane = radius of the sphere

$= \frac{|2\times 1 - 2\times 2 + 1 + 10|}{\sqrt{4+4+1}} = \sqrt{k}$ $\therefore \sqrt{k} = 3$ $\therefore k = 9$

5 (a)

Solution :

$\overrightarrow{A} = (1, 2, 3)$, $\overrightarrow{B} = (2, 3, -5)$

$\therefore \overrightarrow{A}\cdot\overrightarrow{B} = (2 + 6 - 15) = -7 \neq 0$

$\therefore \overrightarrow{A}, \overrightarrow{B}$ are not perpendicular

, $\because \overrightarrow{A}\times\overrightarrow{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 2 & 3 & -5 \end{vmatrix} = -19\hat{i} + 11\hat{j} - \hat{k} \neq \overrightarrow{0}$

$\therefore \overrightarrow{A}, \overrightarrow{B}$ are not parallel

, $\because \overrightarrow{A}\cdot\left(\overrightarrow{A}\times\overrightarrow{B}\right) = (1, 2, 3)\cdot(-19, 11, -1)$
$= -19 + 22 - 3 = 0$

$\therefore \overrightarrow{A} \perp \left(\overrightarrow{A}\times\overrightarrow{B}\right)$

6 (d)

Solution :

$\because \begin{vmatrix} a & b & c \\ d & e & f \\ x & y & z \end{vmatrix} = 2$, $\therefore \begin{vmatrix} 5a & b & c \\ 5d & e & f \\ 5x & y & z \end{vmatrix} = 10$

$\therefore \begin{vmatrix} 5a & b & c \\ 5d & e & f \\ 35x & 7y & 7z \end{vmatrix} = 70$

7 (b)

Solution :

$T_{r+1} = {}^{10}C_r \times \left(\frac{1}{bx}\right)^r \times (ax)^{10-r}$
$= {}^{10}C_r \times \left(\frac{1}{b}\right)^r \times a^{10-r} \times x^{10-2r}$

Put $10 - 2r = 0$ $\therefore r = 5$

$\therefore T_6$ is the term free of x

$\therefore T_6$ = coefficient of T_7 $\therefore \frac{\text{coefficient } T_7}{T_6} = 1$

$\therefore \frac{10-6+1}{6} \times \frac{1}{b} \times \frac{1}{a} = 1$ $\therefore ab = \frac{5}{6}$

8 (c)

Solution :

$7(7x - 6) - 6(14 - 2x) + x(6 - x^2) = 0$

$49x - 42 - 84 + 12x + 6x - x^3 = 0$

$\therefore x^3 - 67x + 126 = 0$

$\because x = -9$ is one of roots of the equation and by using sythetic division.

	1	0	– 67	126
– 9		– 9	81	– 126
	1	– 9	14	0

$\therefore (x + 9)(x^2 - 9x + 14) = 0$

$\therefore (x + 9)(x - 2)(x - 7) = 0$

$\therefore x = -9$, $x = 2$, $x = 7$

$\therefore$ The two other roots are 2 , 7

9 (a)

Solution :

The point B (1 , 0 , 0) lies on the straight line

$\therefore \overrightarrow{BA} = (5, 4, -1) - (1, 0, 0) = (4, 4, -1)$

$\because \vec{d}$ (direction vector of the straight line) = (2 , 9 , 5)

$\therefore \overrightarrow{BA}\times\vec{d} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 4 & -1 \\ 2 & 9 & 5 \end{vmatrix} = 29\hat{i} - 22\hat{j} + 28\hat{k}$

$\therefore$ length of perpendicular $= \frac{\|\overrightarrow{BA}\times\vec{d}\|}{\|\vec{d}\|}$
$= \frac{\sqrt{29^2 + 22^2 + 28^2}}{\sqrt{2^2 + 9^2 + 5^2}}$
$= \sqrt{\frac{2109}{110}}$ length unit.

10 (c)

Solution :

Centre of the sphere M is the point of intersection of medians of the triangle ABC

where A (5 , 5 , 0) , B (0 , 5 , 5) , C (5 , 0 , 5)

$\therefore M\left(\frac{10}{3}, \frac{10}{3}, \frac{10}{3}\right)$

$\therefore$ Radius of the smallest sphere = MA

$= \sqrt{\left(5 - \frac{10}{3}\right)^2 + \left(5 - \frac{10}{3}\right)^2 + \left(0 - \frac{10}{3}\right)^2}$

$= \frac{5\sqrt{6}}{3}$ length unit.

11 (c)

Solution :

$\overrightarrow{A}\cdot\overrightarrow{B} = \|\overrightarrow{A}\|\|\overrightarrow{B}\| \cos\theta = 1 \times 1 \times \cos\theta = \cos\theta$

$\because \cos\theta \in [-1, 1]$ $\therefore \overrightarrow{A}\cdot\overrightarrow{B} \in [-1, 1]$

12 (b)

Solution :

$\because T_4 = T_{11}$

$\therefore {}^{13}C_3 \times \left(\frac{1}{8x}\right)^3 \times (x^2)^{10} = {}^{13}C_{10} \times \left(\frac{1}{8x}\right)^{10} \times (x^2)^3$

$\therefore \left(\frac{1}{3}\right)^3 \times x^{17} = \left(\frac{1}{8}\right)^{10} \times x^{-4}$

$\therefore x^{21} = \left(\frac{1}{8}\right)^7 = \left(\frac{1}{2}\right)^{21}$ $\therefore x = \frac{1}{2}$

13 (c)

Solution :

$A = \begin{pmatrix} 2 & -1 & 3 \\ 4 & -2 & 6 \\ 1 & 0 & 2 \end{pmatrix}$, $|A| = 0$

, $\begin{vmatrix} 2 & -1 \\ 1 & 0 \end{vmatrix} \neq 0$ $\therefore$ RK (A) = 2

, $\because$ the equations are homogeneous

$\therefore$ RK (A) = RK (A^*) = 2 < the number of unknowns.

$\therefore$ The system has an infinite number of solutions.

14 (a)

Solution :

(1 , 2 , 1) . (3 , – 5 , 4) = 3 – 10 + 4 = – 3 < 0

(0 , – 1 , 2) . (3 , – 5 , 4) = 0 + 5 + 8 = 13 > 0

(2 , – 3 , 1) . (3 , – 5 , 4) = 6 + 15 + 4 = 25 > 0

$(1, 2, 4) \cdot (3, -5, 4) = 3 - 10 + 16 = 9 > 0$

$\therefore$ all the points lie on the same side of the plane except the point $(1, 2, 1)$

15 (d)

Solution :

The angle between the two vector $\vec{A}, \vec{B}$ is acute if $\vec{A} \cdot \vec{B} > 0$

$\therefore (x, -3, -1) \cdot (2x, x, -1) > 0$

$\therefore 2x^2 - 3x + 1 > 0$

$\therefore (2x - 1)(x - 1) > 0$

$x \in \mathbb{R} - \left[\frac{1}{2}, 1\right] \qquad \therefore x = 3$

16 (b)

Solution :

$z + 2 = i z - 2i$

$\therefore z(1 - i) = -2 - 2i \quad \left(\text{multiply by } (1+i)\right)$

$\therefore z(1 + 1) = -2(1 + i)^2 = -2(1 + 2i - 1)$

$\therefore 2z = -4i \qquad \therefore z = -2i$

$\therefore$ The modulus $= \sqrt{(-2)^2} = 2$, the amplitude $\theta = \frac{-\pi}{2}$

$\therefore z = 2\left(\cos\frac{-\pi}{2} + i \sin\frac{-\pi}{2}\right)$

17 (b)

Solution :

In order to the intersection line of the two planes intersect z-axis, then the point $(0, 0, z_1)$ lies on each of the two planes

$\therefore 2z_1 + 6 = 0 \qquad \therefore z_1 = -3 \qquad (1)$

$, -z_1 + c = 0 \qquad \therefore z_1 = c \qquad (2)$

From (1), (2) : $\qquad \therefore C = -3$

18 (a)

Solution :

$$\frac{\underline{|n-1}}{n\,\underline{|n-1}} + \frac{\underline{|n+1}}{(n+2)\,\underline{|n+1}} = \frac{6}{n(n+2)}$$

$\frac{1}{n} + \frac{1}{n+2} = \frac{6}{n(n+2)}$ (Multiplying by $n(n+2)$)

$n + 2 + n = 6 \qquad \therefore 2n = 4 \qquad \therefore n = 2$

19 (c)

Solution :

$\vec{d}$ (direction vector of the straight line) $= (2, 1, -1)$

, $\vec{n}$ (the normal direction vector of the plane) $= (1, 1, 0)$

$$\therefore \cos\theta = \frac{|(2, 1, -1) \cdot (1, 1, 0)|}{\sqrt{4+4+1} \times \sqrt{1+1+0}} = \frac{3}{\sqrt{6} \times \sqrt{2}} = \frac{\sqrt{3}}{2}$$

$\therefore \theta = 30$

$\therefore$ The measure of the angle between the line and the plane $= 90° - 30° = 60°$

20 (d)

Solution :

$A = I \qquad \therefore A = A^T = A^{adj} = A^{-1} = I$

21 (a)

Solution :

$z = 2(-i) \qquad \therefore z = 2e^{\frac{-\pi}{2} i}$

$\therefore z^{\frac{1}{2}} = \sqrt{2}\, e^{\frac{\frac{\pi}{2} + 2\pi k}{2} i}$ where $k = 0, 1$

at $k = 0 \qquad \therefore z^{\frac{1}{2}} = \sqrt{2}\, e^{\frac{-\pi}{4} i}$

at $k = 1 \qquad \therefore z^{\frac{1}{2}} = \sqrt{2}\, e^{\frac{3\pi}{4} i}$

22 (d)

Solution :

${}^{x+y}P_2 = {}^{15}P_2 \qquad \therefore x + y = 15 \qquad (1)$

${}^{y-3}C_3 = {}^{7}C_3 \qquad \therefore y - 3 = 7 \qquad \therefore y = 10$

by substitution in (1) : $\therefore x = 5$

$\therefore \underline{|2x - y} = \underline{|10 - 10} = \underline{|0} = 1$

23 (d)

Solution :

$\left(2 + \frac{3}{\omega}\right)\left(2 + \frac{2}{\omega^2}\right)\left(3 - \frac{2}{\omega}\right)\left(3 - \frac{2}{\omega^2}\right)$

$= (2 + 3\omega^2)(2 + 3\omega)(3 - 2\omega^2)(3 - 2\omega)$

$= \left(4 + 6(\omega^2 + \omega) + 9\right)\left(9 - 6(\omega^2 + \omega) + 4\right)$

$= (13 - 6)(13 + 6) = 7 \times 19 = 133$

24 (c)

Solution :

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & x+1 & 1 \\ 1 & 1 & y+1 \end{vmatrix} \text{"doing } C_2 - C_1 \text{ and } C_3 - C_1\text{"}$$

$$= \begin{vmatrix} 1 & 0 & 0 \\ 1 & x & 0 \\ 1 & 0 & y \end{vmatrix} = xy + \text{zero} = xy$$

25 (b)

Solution :

$(1 + 3x + 3x^2 + x^3)^{10} = \left((1 + x)^3\right)^{10} = (1 + x)^{30}$

$T_{r+1} = {}^{30}C_r\, x^r$

Put $r = 4$

$\therefore T_5 = {}^{30}C_4\, x^4$

$\therefore$ Coefficient of $x^4 = {}^{30}C_4$

Exam 12

1 (b)

Solution :

$z_1 z_2 = r_1 r_2 \left(\cos(\theta_1 + \theta_2) + i \sin(\theta_1 + \theta_2)\right)$

$= r_1 r_2 (\cos\pi + i \sin\pi) = r_1 r_2 (-1 + i \times 0) = -r_1 r_2$

2 (b)

Solution :

$$\sum_{r=1}^{6} (1 + \omega^r) = \sum_{r=1}^{6} 1 + \sum_{r=1}^{6} \omega^r = 6 + \frac{\omega(\omega^6 - 1)}{(\omega - 1)}$$

$$= 6 + \frac{\omega \times 0}{\omega - 1} = 6$$

3 (d)

Solution :

$$\because \begin{vmatrix} 1 & 1 & 3 \\ -1 & 0 & 2 \\ 2 & -2 & 1 \end{vmatrix} = 1(0 + 4) - (-1 - 4) + 3(2 - 0)$$

$$= 4 + 5 + 6 = 15 \neq 0$$

$\therefore RK(A) = 3$

$\therefore RK(A^T) = RK(A) = 3$

4 (c)

Solution :

$T_{r+1} = x^3 \times {}^7C_r \times x^r = {}^7C_r \times x^{r+3}$

Put : $r + 3 = 4 \qquad \therefore r = 1$

$T_2 = {}^7C_1\, x^4$

$\therefore$ Coefficient the term has $x^4 = {}^7C_1$

5 (d)

Solution :

To get the smallest sphere passing through three non-collinear points, then all of them lie on the greatest circle in the sphere and make an equilateral triangle of side length $= 4\sqrt{2}$

$\therefore$ The radius length of the circumcircle

$= \frac{4\sqrt{2}}{2 \sin 60°} = \frac{4\sqrt{6}}{3}$

Its centre is $= \left(\frac{4+0+0}{3}, \frac{0+4+0}{3}, \frac{0+0+4}{3}\right)$

$= \left(\frac{4}{3}, \frac{4}{3}, \frac{4}{3}\right)$

$\therefore$ The equation is

$\left(x - \frac{4}{3}\right)^2 + \left(y - \frac{4}{3}\right)^2 + \left(z - \frac{4}{3}\right)^2 = \frac{32}{3}$

6 (b)

Solution :

$$\because z = \frac{5 - 3\sqrt{3}\,i}{1 + 2\sqrt{3}\,i} \times \frac{1 - 2\sqrt{3}\,i}{1 - 2\sqrt{3}\,i} = \frac{-13 - 13\sqrt{3}\,i}{13}$$

$$= -1 - \sqrt{3}\,i$$

$\therefore r = \sqrt{(-1)^2 + \left(-\sqrt{3}\right)^2} = 2$

$, \because x < 0, y < 0$

$\therefore \theta = -\pi + \tan^{-1}\left(\frac{-\sqrt{3}}{-1}\right) = \frac{-2\pi}{3}$

$\therefore z = 2\left(\cos\frac{-2\pi}{3} + i \sin\frac{-2\pi}{3}\right)$

7 (c)

Solution :

$\because$ The equations of the two planes are

$x + y + z = 6 \; (1), \; 2x + 3y + 4z = -5 \qquad (2)$

$, \because \frac{1}{2} \neq \frac{1}{3}$

$\therefore$ The planes are intersecting

multiply (1) by – 3 and adding (2) to it

$\therefore -x + z = -23 \qquad \therefore x = z + 23 \qquad (3)$

Multiply (1) : by – 4 and adding (2) to it

$\therefore -2x - y = -29 \qquad \therefore x = \frac{-y+29}{2} \qquad (4)$

From (3) , (4) :

$\therefore$ The equation of the intersection line is

$x = \frac{-y+29}{2} = z + 23$

8 (d)

Solution :

$$\begin{vmatrix} a+b & 5 & c \\ b+c & 5 & a \\ a+c & 5 & b \end{vmatrix} = 5\begin{vmatrix} a+b & 1 & c \\ b+c & 1 & a \\ a+c & 1 & b \end{vmatrix}$$

$$«C_3 + C_1» = 5\begin{vmatrix} a+b+c & 1 & c \\ a+b+c & 1 & a \\ a+b+c & 1 & b \end{vmatrix}$$

$$= 5(a+b+c)\begin{vmatrix} 1 & 1 & c \\ 1 & 1 & a \\ 1 & 1 & b \end{vmatrix}$$

$= 5(a+b+c) \times \text{zero} = \text{zero}$

9 (c)

Solution :

$\because T_7 = {}^nC_6\, x^{3(n-6)} \left(\frac{5}{x}\right)^6 = {}^nC_6 \times 5^6 \times x^{3n-18-6}$

$, \because T_7$ free of $x \qquad \therefore 3n - 24 = 0$

$\therefore 3n = 24 \qquad \therefore n = 8$

10 (b)

Solution :

$\frac{2}{4} = \frac{k}{6} \qquad \therefore k = 3$

11 (b)

Solution :

$$\because A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 1 & 1 & 0 \end{pmatrix} \qquad \therefore |A| = -1$$

$$, \text{Adj}(A) = \begin{pmatrix} 1 & -1 & -1 \\ 2 & -2 & -1 \\ -2 & 1 & 1 \end{pmatrix}^t = \begin{pmatrix} 1 & 2 & -2 \\ -1 & -2 & 1 \\ -1 & -1 & 1 \end{pmatrix}$$

$$, A^{-1} = \frac{1}{-1}\begin{pmatrix} 1 & 2 & -2 \\ -1 & -2 & 1 \\ -1 & -1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & -2 & 2 \\ 1 & 2 & -1 \\ 1 & 1 & -1 \end{pmatrix}$$

$$\therefore \begin{pmatrix} b \\ c \\ d \end{pmatrix} = \begin{pmatrix} -1 & -2 & 2 \\ 1 & 2 & -1 \\ 1 & 1 & -1 \end{pmatrix}\begin{pmatrix} 5 \\ 1 \\ 7 \end{pmatrix} = \begin{pmatrix} 7 \\ 0 \\ -1 \end{pmatrix}$$

S.S. $= \{(7 , 0 , -1)\}$

12 (a)

Solution :

$$\cos\theta = \frac{|(3 , 1 , -1) \cdot (1 , 4 , -2)|}{\sqrt{9+1+1} \times \sqrt{1+16+4}} = \frac{|3+4+2|}{\sqrt{11}\sqrt{21}} = \frac{9}{\sqrt{231}}$$

$\therefore \theta = \cos^{-1}\frac{9}{\sqrt{231}}$

13 (b)

14 (c)

15 (c)

Solution :

$\because (1+x)^n = 1 + {}^nC_1\, x + {}^nC_2\, x^2 + \ldots\ldots + {}^nC_n\, x^n$

by differentiation the two sides with respect to x

$\therefore n(1+x)^{n-1} = 0 + {}^nC_1 + 2\,{}^nC_2\, x + 3\,{}^nC_3\, x^2 + \ldots\ldots + n\,{}^nC_n\, x^{n-1}$

Put $x = 1$ in both sides

$\therefore {}^nC_1 + 2\,{}^nC_2 + 3\,{}^nC_3 + \ldots\ldots + n\,{}^nC_n = n(2)^{n-1}$

16 (c)

Solution :

$\because (2 , -1 , 1)$ is the normal direction vector of the plane

$\therefore$ The equation of straight line passes through the point A $(1 , 3 , 4)$ and is perpendicular to the plane is $\vec{r} = (1 , 3 , 4) + t(2 , -1 , 1)$

$\therefore \grave{A}$ the image of the point A by reflection in the plane $\in$ the straight line

i.e. $\grave{A} = (1 + 2t , 3 - t , 4 + t)$

$, \because \grave{A}$ is the image of A by reflection in the plane

$\therefore$ The midpoint of $\overline{A\grave{A}}$ lies on the plane

$\therefore \left(\frac{1+1+2t}{2} , \frac{3+3-t}{2} , \frac{4+4+t}{2}\right) \in$ the plane

i.e. $\left(1 + t , 3 - \frac{t}{2} , 4 + \frac{t}{2}\right) \in$ the plane

$\therefore 2(1+t) - \left(3 - \frac{t}{2}\right) + \left(4 + \frac{t}{2}\right) + 3 = 0$

$\therefore 6 + 3t = 0 \qquad \therefore t = -2$

$\therefore \grave{A} = (-3 , 5 , 2)$

17 (b)

Solution :

$\overrightarrow{AB} = \vec{B} - \vec{A} = (4 , -4 , -2) - (6 , 1 , -3)$
$= (-2 , -5 , 1)$

The work done with $\vec{F} = \vec{F} \cdot \overrightarrow{AB}$

$= (1 , -3 , 5) \cdot (-2 , -5 , 1)$

$= 18$ work unit.

18 (d)

19 (b)

Solution :

$\overrightarrow{d_1} = (-1 , 3 , 2) \quad , \quad \overrightarrow{d_2} = (2 , k , m)$

$\because$ The two straight lines are perpendicular

$\therefore \overrightarrow{d_1} \cdot \overrightarrow{d_2} = 0$

$\therefore (-1 , 3 , 2) \cdot (2 , k , m) = 0$

$\therefore -2 + 3k + 2m = 0 \qquad \therefore 3k + 2m = 2$

20 (c)

Solution :

The ordened of the middle term in

the expansion $= \frac{10+2}{2} = 6$

$\therefore T_6 = \frac{28}{27} \qquad \therefore {}^{10}C_5 \left(\frac{1}{2x}\right)^5 (x^2)^5 = \frac{28}{27}$

$\therefore 252 \times \frac{1}{32} \times x^5 = \frac{28}{27}$

$x^5 = \frac{32}{243} = \left(\frac{2}{3}\right)^5 \qquad \therefore x = \frac{2}{3}$

21 (a)

Solution :

$a^2 + b^2 = (2\omega - 3\omega^2)^2 + (3 + 5\omega^2)^2$

$= 4\omega^2 - 12\omega^3 + 9\omega^4 + 9 + 30\omega^2 + 25\omega^4$

$= 4\omega^2 - 12 + 9\omega + 9 + 30\omega^2 + 25\omega$

$= 34\omega^2 + 34\omega - 3$

$= 34(\omega^2 + \omega) - 3 = -34 - 3 = -37$

22 (b)

Solution :

$z = \cos 60° + i \sin 60° = \frac{1}{2} + \frac{\sqrt{3}}{2} i$

$z + 1 = \frac{3}{2} + \frac{\sqrt{3}}{2} i$

$\therefore |z + 1| = \sqrt{\frac{9}{4} + \frac{3}{4}} = \sqrt{3}$

$\therefore \theta = \tan^{-1}\left(\frac{\sqrt{3}}{3}\right) = \frac{\pi}{6}$

$z + 1 = \sqrt{3}\, e^{\frac{\pi}{6} i}$

23 (b)

Solution :

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{vmatrix} \text{ «by add } (C_3 + C_2) \text{ to } C_1\text{»}$$

$$= \begin{vmatrix} 3 & 1 & 1 \\ 1+\omega+\omega^2 & \omega & \omega^2 \\ 1+\omega+\omega^2 & \omega^2 & \omega \end{vmatrix} = \begin{vmatrix} 3 & 1 & 1 \\ 0 & \omega & \omega^2 \\ 0 & \omega^2 & \omega \end{vmatrix}$$

«by multiplying $R_2 \times (-\omega)$ and add it to R_3»

$$= \begin{vmatrix} 3 & 1 & 1 \\ 0 & \omega & \omega^2 \\ 0 & 0 & \omega - \omega^3 \end{vmatrix} = 3 \times \omega \times (\omega - 1)$$

$= 3(\omega^2 - \omega) = 3 \times \pm\sqrt{3}\, i = \pm 3\sqrt{3}\, i$

24 (d)

Solution :

$\cos^2 30° + \cos^2 70° + \cos^2\theta = 1$

$\therefore \cos\theta = 0.365 \qquad \therefore \theta = 68.6°$

25 (c)

Solution :

The term which has the greatest coefficient in the expansion is the middle term.

The order of middle term $= \frac{10+2}{2} = 6$

$\therefore T_6$ which has greatest coefficient in the expansion.

Exam 13

1 (a)

Solution :

$\because \frac{3}{3} = \frac{12}{12} = \frac{-4}{-4} \neq \frac{9}{-17}$

$\therefore$ The two planes are parallel and not coincident

, put $x = -1$, $y = 1$ in equation of the first plane

$\therefore -3 + 12 - 4z = 9 \qquad \therefore z = 0$

$\therefore$ the point $(-1, 1, 0)$ lies on the first plane

$\therefore$ the length of perpendicular line segment between the two planes $= \dfrac{|3 \times (-1) + 12 \times 1 + -4 \times 0 + 17|}{\sqrt{3^2 + 12^2 + (-4)^2}}$

$= \dfrac{26}{13} = 2$ length unit.

---

2 (a)

Solution :

let D is midpoint of $\overline{BC}$ $\qquad \therefore D(1, 1, 1)$

$\therefore$ length of $\overline{AD} = \sqrt{(1-1)^2 + (2-1)^2 + (3-1)^2}$

$= \sqrt{5}$ length unit.

---

3 (c)

---

4 (c)

Solution :

$T_{r+1} = T_{p+1} \times T_{q+1} \quad = {}^4C_p (-x)^p \times {}^9C_q (x)^q$

$= {}^4C_p \times {}^9C_q (-1)^p \times x^{p+q}$

put $p + q = 7$

p	0	1	2	3	4
q	7	6	5	4	3

$\therefore$ coefficient of $x^7 = {}^4C_0 \times {}^9C_7 - {}^4C_1 \times {}^9C_6$

$+ {}^4C_2 \times {}^9C_5 - {}^4C_3 \times {}^9C_4$

$+ {}^4C_4 \times {}^9C_3 = 36$

---

5 (b)

Solution :

$\left(1 + 2\omega^5 + \frac{1}{\omega^2}\right)\left(1 + 2\omega + \frac{1}{\omega^4}\right)$

$= (1 + 2\omega^2 + \omega)(1 + 2\omega + \omega^2) = \omega^2 \times \omega = 1$

---

6 (b)

Solution :

let $|z_1| = |z_2| = r$

$\because$ amplitude (z_1) + amplitude $(z_2) = \pi$

$\therefore z_2 = r(\cos\theta + i\sin\theta)$

$z_1 = r\left(\cos(180° - \theta) + i\sin(180° - \theta)\right)$

$= r(-\cos\theta + i\sin\theta) = -\overline{z_2}$

---

7 (d)

Solution :

$\because$ The direction vector of the straight line

$= (-1, 4, 1) - (1, 2, 2) = (-2, 2, -1)$

$\therefore$ The vector form is $\vec{r} = (2, -1, 1) + t(-2, 2, -1)$

and the Paramatric equations :

$x = 2 - 2t$, $y = -1 + 2t$, $z = 1 - t$

and the Cartesian equation is $\dfrac{x-2}{-2} = \dfrac{y+1}{2} = \dfrac{z-1}{-1}$

---

8 (b)

Solution :

Put $x = 2$

$\therefore$ The value of the determinant = zero

$\therefore \begin{vmatrix} 3 & 1 & -3 \\ 2 & 5 & 1 \\ 1 & -4 & k+2 \end{vmatrix} = 0$

$\therefore 3(5k + 10 + 4) - 1(2k + 4 - 1) - 3(-8 - 5) = 0$

$\therefore 13k = -78 \qquad \therefore k = -6$

---

9 (d)

Solution :

$\overrightarrow{AC} = (4, 0, 0) - (0, 8, 6) = (4, -8, -6)$

, $\overrightarrow{AD} = (0, 0, 6) - (0, 8, 6) = (0, -8, 0)$

$\therefore \overrightarrow{AC} \times \overrightarrow{AD} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -8 & -6 \\ 0 & -8 & 0 \end{vmatrix} = -48\hat{i} - 32\hat{k}$

---

10 (a)

Solution :

$\because (a - x)^{14} = a^{14} - {}^{14}C_1 x a^{13} + {}^{14}C_2 x^2 a^{12} - \dots + x^{14}$

By comparing the two expansions

$\therefore C_2 = {}^{14}C_2 a^{12}$, $C_3 = -{}^{14}C_3 a^{11}$, $C_4 = {}^{14}C_4 a^{10}$

, $\because 4C_4 + 11(C_3 + C_2) = 0$

$\therefore 4 \times {}^{14}C_4 a^{10} - 11 \times {}^{14}C_3 a^{11} + 11 \times {}^{14}C_2 a^{12} = 0$

(by dividing by ${}^{14}C_3 a^{10}$)

$\therefore 11 - 11a + \frac{11}{4}a^2 = 0$ (by multipling by $\frac{4}{11}$)

$\therefore 4 - 4a + a^2 = 0 \qquad \therefore (2 - a)^2 = 0 \qquad \therefore a = 2$

11 (a)

Solution :

$\vec{A} \cdot \vec{B} \times \vec{C}$ = zero

$\therefore \begin{vmatrix} 1 & -2 & 1 \\ m & -5 & 3 \\ 5 & -9 & 4 \end{vmatrix}$ = zero

$\therefore 1(-20 + 27) + 2(4m - 15) + 1(-9m + 25) = 0$

$\therefore 7 + 8m - 30 - 9m + 25 = 0$

$\therefore m = 2$

---

12 (b)

Solution :

$C = A^{-1} B$

$= \dfrac{1}{\begin{vmatrix} 2 & 5 \\ 5 & 3 \end{vmatrix}} \begin{pmatrix} 3 & -5 \\ -5 & 2 \end{pmatrix} \begin{pmatrix} -7 & 3 \\ -46 & 17 \end{pmatrix}$

$= \dfrac{-1}{19} \begin{pmatrix} 209 & -76 \\ -57 & 19 \end{pmatrix} = \begin{pmatrix} -11 & 4 \\ 3 & -1 \end{pmatrix}$

---

13 (d)

Solution :

$\Delta ABC : \dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C} = 2r$

$\therefore \begin{vmatrix} a & b & c \\ 5 & 7 & 8 \\ \sin A & \sin B & \sin C \end{vmatrix} = \begin{vmatrix} 2r\sin A & 2r\sin B & 2r\sin C \\ 5 & 7 & 8 \\ \sin A & \sin B & \sin C \end{vmatrix}$

$= 2r \begin{vmatrix} \sin A & \sin B & \sin C \\ 5 & 7 & 8 \\ \sin A & \sin B & \sin C \end{vmatrix} = 2r \times \text{zero} = \text{zero}$

---

14 (b)

Solution :

$A = \begin{pmatrix} 2 & 5 & 0 \\ 3 & 0 & -1 \\ 0 & 2 & -3 \end{pmatrix} \qquad |A| = \begin{vmatrix} 2 & 5 & 0 \\ 3 & 0 & -1 \\ 0 & 2 & -3 \end{vmatrix}$

$= 2(2) - 5(-9) = 49 \neq 0$

$\therefore$ RK (A) = 3 = No of unknown

$\because$ the equations are homogeneous

$\therefore$ number of solutions = 1

15 (c)

Solution :

$\because (2, 1, 1) \times (3, -2, -1) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 1 \\ 3 & -2 & -1 \end{vmatrix}$

$= \hat{i} + 5\hat{j} - 7\hat{k}$

$\vec{A} = \dfrac{(1, 5, -7)}{\sqrt{1^2 + 5^2 + (-7)^2}} \times \sqrt{75} = (1, 5, -7)$

$\therefore \vec{A} = \hat{i} + 5\hat{j} - 7\hat{k}$

---

16 (a)

Solution :

The length of the perpendicular drawn from the centre $(3, -2, -1)$ to the plane whose equation $(z - 5 = 0)$ is $\dfrac{|-1-5|}{\sqrt{(1)^2 + 0 + 0}} = 6$

$\therefore r = 6$ length unit.

$\therefore$ The equation of the sphere is

$(x - 3)^2 + (y + 2)^2 + (z + 1)^2 = 36$

---

17 (d)

Solution :

$z = (x - y)^2 = \left(\sqrt{3} - i - \sqrt{3} - i\right)^2 = (-2i)^2 = -4$

$= 4(\cos\pi + i\sin\pi) = 4e^{\pi i}$

$\therefore \sqrt[3]{z} = \sqrt[3]{4}\, e^{\left(\frac{\pi + 2\pi k}{3}\right) i}$ where $k = 0, 1, -1$

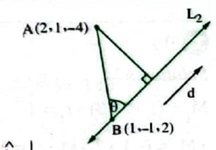
at $k = 0 \qquad \therefore z_1 = \sqrt[3]{4}\, e^{\frac{\pi}{3} i}$

at $k = 1 \qquad \therefore z_2 = \sqrt[3]{4}\, e^{\pi i}$

at $k = -1 \qquad z_3 = \sqrt[3]{4}\, e^{\frac{-\pi}{3} i}$

---

18 (d)



Solution :

Let $A(2, 1, -4) \in L_1$

, $\because B(1, -1, 2) \in L_2$

$\therefore \overrightarrow{AB} \times \vec{d} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & -2 & 6 \\ 2 & 3 & -2 \end{vmatrix} = -14\hat{i} + 10\hat{j} + \hat{k}$

$\therefore$ The distance between the two straight lines

= the length of the perpendicular from A to L_2

$$= \frac{\|\overrightarrow{AB} \times \vec{d}\|}{\|\vec{d}\|} = \frac{\sqrt{(-14)^2+(10)^2+1^2}}{\sqrt{2^2+3^2+(-2)^2}}$$

$\simeq 4.18$ length unit.

---

19 (a)

Solution :

$$A = \begin{pmatrix} 2 & 3 & -1 \\ 1 & -3 & 2 \\ 0 & 1 & 1 \end{pmatrix} \quad \therefore |A| = -14 \neq 0$$

$\therefore$ RK (A) = 3

$\because$ RK (A) = RK (A*) = 3 (number of unknown)

$\therefore$ the system of the equations represent three planes intersected at a point.

---

20 (a)

Solution :

$2 \times {}^nC_4 - 3 \times {}^nC_3 + 2 \times {}^nC_2 = 0$ (Divided by ${}^nC_3$)

$\therefore 2 \frac{{}^nC_4}{{}^nC_3} - 3 + 2 \times \frac{{}^nC_2}{{}^nC_3} = 0$

$\therefore 2 \times \frac{n-4+1}{4} - 3 + 2 \times \frac{3}{n-3+1} = 0$

$\therefore \frac{n-3}{2} - 3 + \frac{6}{n-2} = 0$ (multiplying by $2(n-2)$)

$\therefore (n-3)(n-2) - 6(n-2) + 12 = 0$

$\therefore n^2 - 5n + 6 - 6n + 12 + 12 = 0$

$\therefore n^2 - 11n + 30 = 0$

$\therefore (n-5)(n-6) = 0$

$\therefore n = 5$ or $n = 6$

$\therefore$ S.S. = $\{5, 6\}$

---

21 (b)

Solution :

$M_1 = (1, 0, 3)$, $r_1 = 4$

, $M_2 = (-1, 2, k)$, $r_2 = 5$

$\therefore M_1M_2 = \sqrt{(2)^2+(-2)^2+(3-k)^2}$

$= \sqrt{8+(3-k)^2}$

$\because$ the two spheres are touching externally.

$\therefore M_1M_2 = r_1 + r_2$

$\therefore \sqrt{8+(3-k)^2} = 5 + 4 = 9$

$\therefore 8 + (3-k)^2 = 81 \qquad \therefore (3-k)^2 = 73$

$\therefore 3 - k = \pm\sqrt{73}$

$\therefore k = 3 + \sqrt{73}$ or $3 - \sqrt{73}$

---

22 (b)

Solution :

Number of ways = ${}^6C_3 \times {}^8C_3 = 1120$ ways

---

23 (b)

Solution :

number of terms the expansion

= number of terms of odd ordered terms = $\frac{2n}{2} + 1$

= n + 1 terms

$\therefore n + 1 = 11 \qquad \therefore n = 10$

---

24 (a)

Solution :

$\because {}^nP_2, {}^nP_3, {}^{n+1}P_3$ in arithmetic sequence

$\therefore 2\,{}^nP_3 = {}^nP_2 + {}^{n+1}P_3$

$\therefore 2n(n-1)(n-2) = n(n-1) + (n+1)(n)(n-1)$
divided by $n(n-1)$

$\therefore 2(n-2) = 1 + n + 1 \qquad \therefore 2n - 4 = n + 2$

$\therefore n = 6$

---

25 (d)

Solution :

$z = 2 \times \frac{1}{2} + i \times 1 = 1 + i \qquad \therefore |z| = \sqrt{1+1} = \sqrt{2}$

Amplitude (z) = $\tan^{-1} 1 = \frac{\pi}{4} \qquad \therefore z = \sqrt{2}\, e^{\frac{\pi}{4} i}$

Exam 14

1 (c)

Solution :

The centre of sphere M = (5, −2, 1)

$\because$ M is the midpoint of the diametre $\overline{AB}$

, A (8, −1, 2)

, let B = (x, y, z)

$\therefore \frac{8+x}{2} = 5$, then $x = 2$

, $\frac{-1+y}{2} = -2$, then $y = -3$

, $\frac{2+z}{2} = 1$, then $z = 0$

$\therefore$ B (2, −3, 0)

---

2 (b)

Solution :

$$\begin{vmatrix} 4 & 12 & 4 \\ 8 & -4 & 4 \\ 0 & 16 & 8 \end{vmatrix} = 4 \times 4 \times 4 \times \begin{vmatrix} 1 & 3 & 1 \\ 2 & -1 & 1 \\ 0 & 4 & 2 \end{vmatrix}$$

$= 64\,\Delta$

---

3 (b)

Solution :

${}^{n+2}C_4 = n^2 - 1$

$\therefore \frac{(n+2)\cancel{(n+1)}(n)\cancel{(n-1)}}{\lfloor 4} = \cancel{(n-1)}\cancel{(n+1)}$

$\therefore n^2 + 2n = 24 \qquad \therefore n^2 + 2n - 24 = 0$

$\therefore (n+6)(n-4) = 0 \qquad \therefore n = -6$ (refused)

or $n = 4$

---

4 (a)

Solution :

The order of the middle term = $\frac{8+2}{2} = 5$

$\therefore T_5 = 17920$

$\therefore {}^8C_4 \times \left(\frac{2}{3x}\right)^4 \times (3x^2)^4 = 17920$

$\therefore {}^8C_4 \times \left(\frac{2}{3}\right)^4 \times (3)^4 \times x^4 = 17920$

$\therefore x^4 = 16 \qquad \therefore x = \pm 2$

---

5 (d)

Solution :

$\overrightarrow{d_1} = (-1, -1, 3)$, $\overrightarrow{d_2} = (1, -2, -1)$

$$\overrightarrow{d_1} \times \overrightarrow{d_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & -1 & 3 \\ 1 & -2 & -1 \end{vmatrix} = 7\hat{i} + 2\hat{j} + 3\hat{k}$$

$\because$ the point (−1, 2, 1) belongs to the plane

$\therefore$ equation of the plane is

$(7, 2, 3) \cdot \vec{r} = (7, 2, 3) \cdot (-1, 2, 1)$

$\therefore 7x + 2y + 3z = -7 + 4 + 3$

$\therefore 7x + 2y + 3z = 0$

---

6 (a)

Solution :

$\|\vec{A} \times \vec{B}\|^2 + |\vec{A} \cdot \vec{B}|^2$

$= \|\vec{A}\|^2 \|\vec{B}\|^2 \sin^2\theta + \|\vec{A}\|^2 \|\vec{B}\|^2 \cos^2\theta$

$= \|\vec{A}\|^2 \|\vec{B}\|^2 (\sin^2\theta + \cos^2\theta)$

$= \|\vec{A}\|^2 \|\vec{B}\|^2 = A^2 B^2$

---

7 (b)

Solution :

direction vector of the straight line = $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$

then direction vector is (1, 1, 1)

, $\because$ the point (2, 3, −5) $\in$ straight line.

$\therefore$ equation of the straight line : $\frac{x-2}{1} = \frac{y-3}{1} = \frac{z+5}{1}$

---

8 (d)

Solution :

$z = 2^n \left(\frac{1}{2} + \frac{\sqrt{3}}{2} i\right)^n = n^2 (\cos 60° + i \sin 60°)^n$

, $\because |z| = 8$, $2^n = 8 \qquad \therefore n = 3$

$z = 8(\cos 60° + i \sin 60°)^3$

$= 8(\cos 180° + i \sin 180°)$

$\therefore$ amplitude of (z) = π

---

9 (b)

Solution :

$\because z_1 = 4\left(\cos \frac{-5\pi}{3} + i \sin \frac{-5\pi}{3}\right) = 4 e^{\frac{-5}{3}\pi i}$

, $z_2 = e^{\frac{\pi}{2} i}$

$\therefore z = \frac{z_2}{z_1} = \frac{e^{\frac{\pi}{2} i}}{4 e^{\frac{-5}{3}\pi i}} = \frac{1}{4} e^{2\frac{1}{6}\pi i} = \frac{1}{4} e^{\frac{1}{6}\pi i}$

$= \frac{1}{4}\left(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6}\right)$

10 (c)

Solution :

$\overrightarrow{AB} = (2, 5, -1) - (1, 2, 3) = (1, 3, -4)$

$, \overrightarrow{AC} = (-1, 1, 2) - (1, 2, 3) = (-2, -1, -1)$

$\therefore \overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 3 & -4 \\ -2 & -1 & -1 \end{vmatrix} = -7\hat{i} + 9\hat{j} + 5\hat{k}$

$\|\overrightarrow{AB} \times \overrightarrow{AC}\| = \sqrt{49 + 81 + 25} = \sqrt{155}$

$\therefore$ area of $\Delta ABC = \frac{\sqrt{155}}{2}$ square unit.

---

11 (b)

Solution :

$\because T_{r+1} = {}^nC_r \left(\frac{1}{x}\right)^r (x^2)^{n-r} = {}^nC_r\, x^{2n-3r}$

to find the free term of x, put $2n - 3r = 0$

$\therefore r = \frac{2n}{3}$

$, \because r \in \mathbb{Z}^+$ $\qquad \therefore$ n is a multiple of 3

---

12 (c)

Solution :

$$A = \begin{pmatrix} 3 & 1 & 4 \\ 2 & 3 & 5 \\ -1 & 2 & 1 \end{pmatrix}$$

$\therefore |A| = \begin{vmatrix} 3 & 1 & 4 \\ 2 & 3 & 5 \\ -1 & 2 & 1 \end{vmatrix} = 0, \because \begin{vmatrix} 3 & 1 \\ 2 & 3 \end{vmatrix} \neq 0$

$\therefore RK(A) = RK(A^*) = 2 <$ the number of variables

$\therefore$ The system has an infinite number of solutions one of them is the trivial solution.

---

13 (d)

Solution :

$1 + \omega + \omega^2 + \omega^3 + \dots + \omega^{100} = \frac{1(\omega^{101} - 1)}{(\omega - 1)}$

$= \frac{\omega^2 - 1}{\omega - 1} = \frac{(\omega + 1)\cancel{(\omega - 1)}}{\cancel{(\omega - 1)}} = \omega + 1 = -\omega^2$

---

14 (d)

15 (a)

Solution :

Let $(x_1, y_1, z_1) \in$ the plane P_1

$\therefore$ The distance between the two planes

= the length of the perpendicular drawn from (x_1, y_1, z_1) to the plane P_2

$\therefore$ The distance between the two planes

$= \frac{|a x_1 + b y_1 + c z_1 + d_2|}{\sqrt{a^2 + b^2 + c^2}}$

, from the equation of the 1^{st} plane

$\therefore a x_1 + b y_1 + c z_1 = -d_1$

$\therefore$ The distance between the two planes

$= \frac{|d_2 - d_1|}{\sqrt{a^2 + b^2 + c^2}}$ length unit.

---

16 (b)

Solution :

$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & 7 & 5 \\ -5 & 7 & -3 \end{vmatrix}$

$= -56\hat{i} - 34\hat{j} + 14\hat{k}$

Let the angle between two vectors $(\vec{A} \times \vec{B})$ and $\vec{C}$ is θ

$\therefore \cos\theta = \frac{(-56, -34, 14) \cdot (7, -5, -3)}{\sqrt{4488}\sqrt{83}}$

$= \frac{-392 + 170 - 42}{\sqrt{372504}}$

$\therefore \theta = 115^\circ\, 38'$

---

17 (a)

Solution :

$\overrightarrow{AB}$ (the vector which passes through the two points).

$= (3, -1, 2) - (0, 0, 0) = (3, -1, 2)$

$\vec{C}$ (direction vector of the straight line) $= (1, -4, 7)$

$\therefore \vec{n}$ the normal direction of the plane

$= \overrightarrow{AB} \times \vec{C} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 2 \\ 1 & -4 & 7 \end{vmatrix}$

$= \hat{i} - 19\hat{j} - 11\hat{k}$

$\because$ The required plane passes through the origin point.

$\therefore$ Its equation is $x - 19y - 11z = 0$

---

18 (d)

Solution :

$\text{L.H.S} = \begin{vmatrix} 1 & -2c + 2c^2 & -3c + 3c^2 \\ 1 & 1 + 2c^2 & 3c^2 \\ c & 2c^3 & 1 + 3c^3 \end{vmatrix}$

(by doing $(R_3 - c R_1)$)

$= \begin{vmatrix} 1 & -2c + 2c^2 & -3c + 3c^2 \\ 1 & 1 + 2c^2 & 3c^2 \\ 0 & -c & 1 \end{vmatrix}$

(by doing $(R_2 - R_1)$)

$= \begin{vmatrix} 1 & -2c + 2c^2 & -3c + 3c^2 \\ 0 & 1 + 2c & 3c \\ 0 & -c & 1 \end{vmatrix}$

(by doing $(c_2 + c c_3)$)

$= \begin{vmatrix} 1 & -c^2 + 3c^3 - 2c & -3c + 3c^2 \\ 0 & 1 + 2c + 3c^2 & 3c \\ 0 & 0 & 1 \end{vmatrix}$

$= 1 \times (1 + 2c + 3c^2) \times 1 = 1 + 2c + 3c^2$

---

19 (b)

Solution :

$\because$ the midpoint of $\overline{AB}$ lies on the plane xz

$\therefore$ coordinates of the midpoint of $\overline{AB}$ is $(x, 0, z)$

$\therefore \frac{12 + t + 3t}{2} = 0$ $\qquad \therefore t = -3$

---

20 (b)

Solution :

the vector component of the force $\vec{F}$ in direction of

$\vec{A} = \left(\frac{\vec{F} \cdot \vec{A}}{\|\vec{A}\|^2}\right)\vec{A}$

$= \left(\frac{(1, 2, -4) \cdot (2, 4, -4)}{4 + 16 + 16}\right)(2, 4, -4)$

$= \left(\frac{2 + 8 + 16}{36}\right)(2, 4, -4)$

$= \frac{13}{18}(2, 4, -4) = \frac{13}{9}(1, 2, -2)$

$= \frac{13}{9}(\hat{i} + 2\hat{j} - 2\hat{k})$

21 (c)

Solution :

T_7 from the start $= {}^nC_6 \times \left(\frac{1}{\sqrt[3]{2}}\right)^6 \times \left(\sqrt[3]{2}\right)^{n-6}$

$= {}^nC_6 \left(\sqrt[3]{2}\right)^{n-12}$

T_7 from the end $= {}^nC_6 \times \left(\sqrt[3]{2}\right)^6 \times \left(\frac{1}{\sqrt[3]{2}}\right)^{n-6}$

$= {}^nC_6 \times \left(\sqrt[3]{2}\right)^{12-n}$

$\frac{T_7 \text{ from the start}}{T_7 \text{ from the end}} = \frac{{}^nC_6 \left(\sqrt[3]{2}\right)^{n-12}}{{}^nC_6 \left(\sqrt[3]{2}\right)^{12-n}} = \frac{1}{4}$

$\therefore \left(\sqrt[3]{2}\right)^{2n-24} = \frac{1}{4}$ $\qquad \therefore 2^{\frac{2}{3}n - 8} = 2^{-2}$

$\therefore \frac{2}{3}n - 8 = -2$ $\qquad \therefore \frac{2}{3}n = 6$

$\therefore n = 9$

---

22 (d)

Solution :

$\because z = \frac{-1 + \sqrt{3}\,i}{2}$

$\therefore z_1 = \frac{1 + \frac{1}{2} - \frac{\sqrt{3}}{2}i}{1 - \frac{1}{2} + \frac{\sqrt{3}}{2}i} = \frac{3 - \sqrt{3}\,i}{1 + \sqrt{3}\,i} \times \frac{1 - \sqrt{3}\,i}{1 - \sqrt{3}\,i} = -\sqrt{3}\,i$

$\therefore z_1 = \sqrt{3}(-i) = \sqrt{3}\left(\cos\left(\frac{-\pi}{2}\right) + i\sin\left(\frac{-\pi}{2}\right)\right)$

$= \sqrt{3}\, e^{\frac{-\pi}{2} i}$

$\therefore \sqrt{z_1} = \sqrt[4]{3}\, e^{\left(\frac{\frac{-\pi}{2} + 2\pi n}{2}\right) i}$ where $n = 0, 1$

at $n = 0$ $\qquad \therefore \sqrt{z_1} = \sqrt[4]{3}\, e^{\frac{-\pi}{4} i}$

at $n = 1$ $\qquad \therefore \sqrt{z_1} = \sqrt[4]{3}\, e^{\frac{3}{4} i}$

---

23 (a)

Solution :

$A^{-1} = \frac{1}{|A|} A^{adj}$

$A^{adj} = |A|\, A^{-1}$

---

24 (c)

Solution :

$T_6 + \frac{1}{4} T_7 = 7 \times T_8$ (divide by T_7)

$\therefore \frac{T_6}{T_7} + \frac{1}{4} = 7\frac{T_8}{T_7}$

$\therefore \frac{6}{9-6+1} \times \frac{2}{x} + \frac{1}{4} = 7 \times \frac{9-7+1}{7} \times \frac{x}{2}$

$\therefore \frac{3}{x} + \frac{1}{4} = \frac{3}{2}x$ (multiply by $(4x)$)

$\therefore 12 + x = 6x^2$ $\qquad \therefore 6x^2 - x - 12 = 0$

$\therefore x = \frac{3}{2}$ or $\frac{-4}{3}$

25 (c)

Solution :

$3|x-1|-8=7$ $\qquad \therefore |x-1|=5$

$\therefore x-1=5$ $\qquad \therefore x=6$

or $x-1=-5$ $\qquad \therefore x=-4$

$S.S. = \{6, -4\}$

Exam 15

1 (a)

Solution :

$\because$ Order of the term from starting $= 13-5+1=9$

$\therefore T_5$ from the end $= T_9$ from starting

$$= {}^{12}C_8\left(\frac{-2}{x^2}\right)^8 \times \left(\frac{x^3}{2}\right)^4 = \frac{7920}{x^4}$$

2 (d)

Solution :

$\because \overline{DE} \,//\, \overline{BC}$ $\qquad \therefore \frac{DE}{BC} = \frac{AD}{AB} = \frac{AE}{AC} = k$

$$\therefore \begin{vmatrix} 5 & 6 & 7 \\ k\times BC & k\times AB & k\times AC \\ BC & AB & AC \end{vmatrix} = k\begin{vmatrix} 5 & 6 & 7 \\ BC & AB & AC \\ BC & AB & AC \end{vmatrix}$$

$= k \times \text{zero} = \text{zero}$

3 (a)

Solution :

Number of ways $= {}^4C_1 \times {}^5C_2 = 4 \times {}^5C_2$

4 (a)

Solution :

$\because$ The two straight lines passing through the two points $(1, 1, 0)$

$\therefore$ The two straight lines are intersecting

$\therefore$ The normal direction vector of the plane is

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -1 \\ -1 & 1 & -2 \end{vmatrix} = -3\hat{i} + 3\hat{j} + 3\hat{k}$$

$\therefore$ The equation of the required plane is

$\vec{r} \cdot (-3, 3, 3) = (1, 1, 0) \cdot (-3, 3, 3)$

i.e. $-3x + 3y + 3z = 0$

5 (c)

Solution :

$$\because A = \begin{pmatrix} 1 & -2 & 2 \\ 3 & 0 & 4 \\ 0 & -1 & 6 \end{pmatrix}$$

$$\therefore |A| = \begin{vmatrix} 1 & -2 & 2 \\ 3 & 0 & 4 \\ 0 & -1 & 6 \end{vmatrix} \neq 0$$

$\therefore RK(A) = RK(A^*) = 3 =$ number of variables

$\therefore$ The equations has only the trivial solution.

6 (c)

Solution :

By taking (-1) common factor from each row

$$\therefore \Delta = (-1)^3 \begin{vmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{vmatrix}$$

$\because$ The result determinant is the transpose of the original determinant.

$\therefore \Delta = -\Delta$ $\qquad \therefore 2\Delta = 0 \Rightarrow \Delta = 0$

7 (b)

Solution :

By resolution $\vec{A}$ into two components :

1^{st} component in direction $\overrightarrow{OZ}$ and its magnitude

$A_z = \|\vec{A}\| \cos\theta_z = 10\cos 30° = 5\sqrt{3}$

2^{nd} component in xy – plane and its magnitude A_{xy}

$= \|\vec{A}\| \sin\theta_z = 10 \sin 30° = 5$

, then resolve A_{xy} into two components

1^{st} in direction $\overrightarrow{Ox}$ and its magnitude is

$A_x = A_{xy}\cos 45° = 5 \times \frac{1}{\sqrt{2}} = \frac{5\sqrt{2}}{2}$

2^{nd} in direction $\overrightarrow{Oy}$ and its magnitude

$A_y = A_{xy}\sin 45° = 5 \times \frac{1}{\sqrt{2}} = \frac{5\sqrt{2}}{2}$

$\therefore \vec{A} = \frac{5\sqrt{2}}{2}\hat{i} + \frac{5\sqrt{2}}{2}\hat{j} + 5\sqrt{3}\hat{k}$

To find the direction angles.

$\vec{U}_A = \frac{\vec{A}}{\|\vec{A}\|} = \left(\frac{\sqrt{2}}{4}, \frac{\sqrt{2}}{4}, \frac{\sqrt{3}}{2}\right)$

$\therefore \theta_x = \theta_y = 69° 18', \theta_z = 30°$

8 (d)

Solution :

$$\text{L.H.S.} = \frac{\omega^2}{(1+4\omega^2+4\omega)^2} + \frac{(1+4\omega+4\omega^2)^2}{\omega^2}$$

$$= \frac{\omega^2}{9} + \frac{9}{\omega} = \frac{\omega^3+81}{9\omega} = \frac{82}{9\omega} = \frac{82}{9}\omega^2$$

$\frac{82}{9}\omega^2 = a\,\omega^2$ $\qquad \therefore a = \frac{82}{9}$

9 (a)

Solution :

$\because (1+x+x^2)^6 = \left(1 + x(1+x)\right)^6$

$\therefore T_{r+1} = {}^6C_r\left(x(1+x)\right)^r = {}^6C_r\, x^r (1+x)^r$

In expansion of $(1+x)^r$

$\therefore T_{r+1} = {}^rC_n\, x^n, n \le r$

$T_{r+1} = {}^6C_r\, x^r . {}^rC_n\, x^n = {}^6C_r . {}^rC_n\, x^{r+n}$

Where $n \le r \le 6$ $\qquad \therefore$ Put $r + n = 5$

n	0	1	2
r	5	4	3

coefficient of the expansion $(1+x+x^2)^6$ equals

${}^6C_5 . {}^5C_0 + {}^6C_4 . {}^4C_1 + {}^6C_3 . {}^3C_2$

$= 6 + 60 + 60 = 126$

10 (a)

11 (d)

Solution :

$2\|\vec{A}\times\vec{B}\|(\vec{A}.\vec{B})$

$= 2\|\vec{A}\|\|\vec{B}\|\sin\theta \times \|\vec{A}\|\|\vec{B}\|\cos\theta$

$= 2 \times 1 \times 1 \times \sin\theta \times 1 \times 1 \times \cos\theta$

$= 2\sin\theta\cos\theta = \sin 2\theta$

12 (a)

Solution :

direction vector of the straight line $= \pm(2, -4, 2)$

$\therefore$ The equation of the straight line is :

$\vec{r} = (2, -1, 3) \pm t(2, -4, 2)$

13 (b)

Solution :

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 4 & \ell & -\ell \\ 3 & 2 & -4 \end{pmatrix}$$

$\therefore |A| = 1(-4\ell + 2\ell) - 1(-16 + 3\ell) + 1(8 - 3\ell)$

$= -8\ell + 24$

$\therefore$ The system of the equations has unique solution if :

$-8\ell + 24 \neq 0$

$\therefore \ell \neq 3$ $\qquad \therefore \ell \in \mathbb{R} - \{3\}$

14 (d)

15 (b)

Solution :

Length of perpendicular $= \frac{|2(2) - 2(3) + 1(1) - 5|}{\sqrt{2^2 + (-2)^2 + (1)^2}}$

$= 2$ length unit.

16 (a)

Solution :

$z_1 + z_2 = \cos 75° + i\sin 75° + \cos 15° + i\sin 15°$

$= \cos 75° + \cos 15° + i(\sin 75° + \sin 15°)$

$= \cos(45° + 30°) + \cos(45° - 30°)$

$+ i\left(\sin(45° + 30°) + \sin(45° - 30°)\right)$

$$= (\cos 45° \cos 30° - \sin 45° \sin 30°) + (\cos 45° \cos 30° + \sin 45° \sin 30°) + i\,(\sin 45° \cos 30° + \cos 45° \sin 30° + \sin 45° \cos 30° - \cos 45° \sin 30°)$$

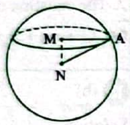
$$= 2 \cos 45° \cos 30° + 2 \sin 45° \cos 30°\, i$$

$$= 2 \times \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + 2 \times \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2}\, i$$

$$= \frac{\sqrt{6}}{2} + \frac{\sqrt{6}}{2}\, i$$

17 (b)

Solution :



Centre of the sphere is $N(0, 0, 0)$ and length of the radius $= 7$ length unit

MN = length of perpendicular segment from N to the plane $= \dfrac{|\text{zero} - 5\sqrt{14}|}{\sqrt{4+9+1}} = 5$ length unit.

$\therefore$ r of the circle $= \sqrt{7^2 - 5^2} = 2\sqrt{6}$ length unit.

18 (b)

Solution :

$\because \overrightarrow{A} \,//\, \overrightarrow{B}$ $\qquad \therefore \dfrac{k}{-2} = \dfrac{3}{9} = \dfrac{-4}{m}$

$\therefore$ km $= 8$

19 (c)

Solution :

$$z = \left(\cos\left(\frac{-\pi}{2} + \theta\right) + i \sin\left(\frac{-\pi}{2} + \theta\right)\right)^n$$

$$= \cos n\left(\frac{-\pi}{2} + \theta\right) + i \sin n\left(\frac{-\pi}{2} + \theta\right)$$

$\therefore$ The principle amplitude of $(z) = \left(\dfrac{-\pi}{2} + \theta\right) n$

20 (d)

Solution :

$|A| = |2B| = 2^3 |B| = 8 \times 5 = 40$

21 (c)

Solution :

$\because (1 - x)^n = {}^nC_0 - {}^nC_1 x + {}^nC_2 x^2 - {}^nC_3 x^3 + \ldots + (-1)^n\, {}^nC_n x^n$

Put $x = 1$

$\therefore {}^nC_0 - {}^nC_1 - {}^nC_2 + {}^nC_3 + \ldots + (-1)^n\, {}^nC_n$

$= (1 - 1)^n =$ zero

22 (c)

23 (d)

Solution :

$$(16 + 32x + 24x^2 + 8x^3 + x^4)^5 = \left((2 + x)^4\right)^5 = (2 + x)^{20}$$

The order of the middle term $= \dfrac{20 + 2}{2} = 11$

Coefficient of the middle term (Coefficient of T_{11})

$= {}^{20}C_{10} \times 2^{10} = 1024 \times {}^{20}C_{10}$

24 (b)

Solution :

$$e^{\theta i} + e^{-\theta i} = (\cos\theta + i \sin\theta) + (\cos(-\theta) + i \sin(-\theta))$$

$$= \cos\theta + i \sin\theta + \cos\theta - i \sin\theta = 2 \cos\theta$$

25 (d)

Solution :

$\because {}^nP_x + {}^xP_n = 1440$ $\qquad \therefore x = n$

$\therefore {}^nP_n + {}^nP_n = 1440$ $\qquad \therefore 2\,{}^nP_n = 1440$

$\therefore {}^nP_n = 720 = {}^6P_6$ $\qquad \therefore n = 6$

$\therefore x = 6$ $\qquad \therefore {}^{n+4}P_{x-5} = {}^{10}P_1 = 10$

Exam 16

1 (b)

Solution :

$r = \sqrt{(-2)^2 + (3)^2 + (-4)^2 - 4} = 5$ length unit.

$\therefore$ diameter $= 2 \times 5 = 10$ length unit.

2 (a)

Solution :

the determinant

$$= \begin{vmatrix} 1 & 1 & 1 \\ n & n+1 & n+2 \\ \frac{1}{2} n(n-1) & \frac{1}{2} n(n+1) & \frac{1}{2}(n+2)(n+1) \end{vmatrix}$$

$$= \frac{1}{2} \begin{vmatrix} 1 & 1 & 1 \\ n & n+1 & n+2 \\ n^2 - n & n^2 + n & n^2 + 3n + 2 \end{vmatrix}$$

(by doing $C_3 - C_2$ then $C_2 - C_1$)

$$= \frac{1}{2} \begin{vmatrix} 1 & 0 & 0 \\ n & 1 & 1 \\ n^2 - n & 2n & 2n + 2 \end{vmatrix} \text{ (by doing } C_3 - C_2\text{)}$$

$$= \frac{1}{2} \begin{vmatrix} 1 & 0 & 0 \\ n & 1 & 0 \\ n^2 - n & 2n & 2 \end{vmatrix} = \frac{1}{2} \times 1 \times 1 \times 2 = 1$$

3 (c)

Solution :

$$T_{r+1} = {}^{11}C_r \times \left(\frac{1}{ax}\right)^r \times (a x^2)^{11-r}$$

$$= {}^{11}C_r \times a^{11-2r} \times x^{22-3r}$$

Put $22 - 3r = 4$ $\qquad \therefore r = 6$

$\therefore T_7$ is the term containing x^4

Put $22 - 3r = 7$ $\qquad \therefore r = 5$

$\therefore T_6$ is the term containing x^7

$\therefore$ Coefficient T_7 = Coefficient T_6

$\therefore \dfrac{\text{Coefficient } T_7}{\text{Coefficient } T_6} = 1$ $\qquad \therefore \dfrac{11 - 6 + 1}{6} \times \dfrac{1}{a} \times \dfrac{1}{a} = 1$

$\therefore \dfrac{1}{a^2} = 1$ $\qquad \therefore a = \pm 1$

4 (c)

Solution :

$\because \overrightarrow{A} + \overrightarrow{B} = \overrightarrow{C}$ $\qquad \therefore \|\overrightarrow{A} + \overrightarrow{B}\|^2 = \|\overrightarrow{C}\|^2$

$\|\overrightarrow{A}\|^2 + \|\overrightarrow{B}\|^2 + 2(\overrightarrow{A} \cdot \overrightarrow{B}) = \|\overrightarrow{C}\|^2$

$\therefore 16 + 36 + 2(\overrightarrow{A} \cdot \overrightarrow{B}) = 64$

$\overrightarrow{A} \cdot \overrightarrow{B} = 6$ $\qquad \therefore \cos\theta = \dfrac{\overrightarrow{A} \cdot \overrightarrow{B}}{\|\overrightarrow{A}\| \|\overrightarrow{B}\|} = \dfrac{6}{4 \times 6} = \dfrac{1}{4}$

$\theta = \cos^{-1}\left(\dfrac{1}{4}\right)$

5 (d)

Solution :

$$\overrightarrow{n_1} \times \overrightarrow{n_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 1 & -3 & -1 \end{vmatrix} = 4\hat{i} + 3\hat{j} - 5\hat{k}$$

$\therefore$ direction vector of intersection line of the two planes $= (4, 3, -5)$

$\therefore$ The point $(1, 1, 0)$ satisfies both of equations the two planes

$\therefore$ The equation of intersection line is

$$\frac{x - 1}{4} = \frac{y - 1}{3} = \frac{z}{-5}$$

6 (c)

7 (a)

Solution :

$$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{\begin{vmatrix} 2 & -1 \\ 1 & 2 \end{vmatrix}} \times \begin{pmatrix} 2 & 1 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} 3 \\ 4 \end{pmatrix} = \frac{1}{5} \begin{pmatrix} 10 \\ 5 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

8 (b)

Solution :

Number of ways $= (n - r + 1)\, \underline{|4} = (10 - 4 + 1)\, \underline{|4} = 7\, \underline{|4}$

9 (a)

10 (b)

Solution :

$\dfrac{{}^{n+2}P_r}{{}^{n+2}C_r} = 2$ $\qquad \therefore r = 2$

$\because \dfrac{{}^nC_{r+1}}{{}^nC_r} = \dfrac{n - (r+1) + 1}{r + 1} = \dfrac{5}{3}$ $\qquad \therefore \dfrac{n - 2}{3} = \dfrac{5}{3}$

$\therefore n = 7$ $\qquad \therefore {}^{2n}C_{n-r} = {}^{14}C_5$

11 (c)

Solution :

$$T_{r+1} = {}^{10}C_r \left(\frac{1}{bx}\right)^r (a x)^{10-r} = {}^{10}C_r\, b^{-r} a^{10-r} x^{10-2r}$$

to find the free term of x, put $10 - 2r = 0$

$\therefore r = 5$ $\qquad \therefore$ free term of x is T_6

$\because T_6 =$ coeff. T_7 $\qquad \therefore \dfrac{\text{coeff. } T_7}{T_6} = 1$

$\because \dfrac{\text{coeff. } T_7}{T_6} = \dfrac{10 - 6 + 1}{6} \left(\dfrac{1}{b\,a}\right)$ $\qquad \therefore \dfrac{5}{6} \times \dfrac{1}{a\,b} = 1$

$\therefore ab = \dfrac{5}{6}$

12 (b)

Solution :

$$\because \overrightarrow{A} \cdot \overrightarrow{B} \times \overrightarrow{C} = \begin{vmatrix} 3 & -4 & 1 \\ 0 & 2 & -3 \\ 3 & 2 & 2 \end{vmatrix} = 60$$

$\therefore$ Volume of the parallelepiped $= |60| = 60$ volume unit.

13 (d)

Solution :

$$\text{L.H.S.} = \begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ac & bc & c^2+1 \end{vmatrix}$$

$$= \begin{vmatrix} a^2 & ab & ac \\ ab & b^2+1 & bc \\ ac & bc & c^2+1 \end{vmatrix} + \begin{vmatrix} 1 & ab & ac \\ 0 & b^2+1 & bc \\ 0 & bc & c^2+1 \end{vmatrix}$$

taking (a) common factor from R_1 , C_1 in 1^{st} det.

$$= a^2 \begin{vmatrix} 1 & b & c \\ b & b^2+1 & bc \\ c & bc & c^2+1 \end{vmatrix} + \begin{vmatrix} 1 & ab & ac \\ 0 & b^2 & bc \\ 0 & bc & c^2+1 \end{vmatrix} + \begin{vmatrix} 1 & 0 & ac \\ 0 & 1 & bc \\ 0 & 0 & c^2+1 \end{vmatrix}$$

(by doing $R_2 - b R_1$, $R_3 - c R_1$) in the 1^{st} det.

, (b common factor from R_2 , C_2 in the 2^{nd} det.)

$$= a^2 \begin{vmatrix} 1 & b & c \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} + b^2 \begin{vmatrix} 1 & a & ac \\ 0 & 1 & c \\ 0 & c & c^2+1 \end{vmatrix} + c^2 + 1$$

by doing $(R_3 - c R_2)$ in the 2^{nd} det.

$$= a^2 + b^2 \begin{vmatrix} 1 & a & ac \\ 0 & 1 & c \\ 0 & 0 & 1 \end{vmatrix} + c^2 + 1 = a^2 + b^2 + c^2 + 1$$

$\therefore \ell = 1$

14 (a)

Solution :

$\overrightarrow{d_1} = (2 , -1 , 1)$, $\overrightarrow{d_2} = (-2 , 7 , 11)$

$\therefore \overrightarrow{d_1} \cdot \overrightarrow{d_2} = (2 , -1 , 1) \cdot (-2 , 7 , 11)$

$= 2 \times -2 + (-1) \times 7 + 1 \times 11 = \text{zero}$

$\therefore$ The two straight lines are perpendicular

, $\therefore$ at the point of intersection $\overrightarrow{r_1} = \overrightarrow{r_2}$

$\therefore (1 , 2 , 4) + t_1 (2 , -1 , 1)$

$= (1 , 1 , 1) + t_2 (-2 , 7 , 11)$

$\therefore 1 + 2 t_1 = 1 - 2 t_2 \qquad \therefore t_1 + t_2 = 0 \qquad (1)$

, $2 - t_1 = 1 + 7 t_2$

$\therefore t_1 + 7 t_2 = 1 \qquad (2)$

, $4 + t_1 = 1 + 11 t_2 \qquad \therefore t_1 - 11 t_2 = -3 \qquad (3)$

from (1) , (2) we get : $t_1 = \frac{-1}{6}$, $t_2 = \frac{1}{6}$

By substitute in (3) : $\frac{-1}{6} - 11 \times \frac{1}{6} \neq -3$

i.e. These values don't satisfy equation (3)

$\therefore$ The two straight lines are skew.

15 (c)

Solution :

$$z = \frac{2(5 - 3\sqrt{3}\,i)}{1 + 2\sqrt{3}\,i} \times \frac{1 - 2\sqrt{3}\,i}{1 - 2\sqrt{3}\,i} = -2 - 2\sqrt{3}\,i$$

$\therefore |z| = \sqrt{4 + 12} = 4$

$\because x < 0 , y < 0 \qquad \therefore \theta$ lies in the third quadrant.

$$\therefore \theta = -\pi + \tan^{-1}\left(\frac{-2\sqrt{3}}{-2}\right) = -\pi + \frac{\pi}{3} = \frac{-2\pi}{3}$$

$$\therefore z = 4\left(\cos\frac{-2\pi}{3} + i \sin\frac{-2\pi}{3}\right) = 4\, e^{\frac{-2\pi}{3} i}$$

$$\therefore \sqrt{z} = 2\, e^{\left(\frac{\frac{-2\pi}{3} + 2\pi n}{2}\right) i}$$

Where n = 0 , 1

At n = 0 :

$\therefore$ The first square root of the number $z = 2\, e^{\frac{-\pi}{3} i}$

At n = 1 :

$\therefore$ The second square root of the number $z = 2\, e^{\frac{2\pi}{3} i}$

16 (b)

Solution :

$\because 3x + 2y + 4z = 12$ (Divided by 12)

$\frac{x}{4} + \frac{y}{6} + \frac{z}{3} = 1$

$\therefore A = (4 , 0 , 0)$

$B = (0 , 6 , 0)$

$C = (0 , 0 , 3)$

$\therefore \overrightarrow{AB} = (-4 , 6 , 0)$, $\overrightarrow{AC} = (-4 , 0 , 3)$

$$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -4 & 6 & 0 \\ -4 & 0 & 3 \end{vmatrix} = 18\hat{i} + 12\hat{j} + 24\hat{k}$$

$\therefore \| \overrightarrow{AB} \times \overrightarrow{AC} \| = \sqrt{18^2 + 12^2 + 24^2} = 6\sqrt{29}$

Area of Δ ABC $= \frac{1}{2} \| \overrightarrow{AB} \times \overrightarrow{AC} \| = 3\sqrt{29}$ square unit.

17 (d)

Solution :

Order of the two middle terms are

$\frac{(2n+1)+1}{2}$, $\frac{(2n+1)+3}{2}$

i.e : They are n + 1 , n + 2

$\because T_{n+1} = T_{n+2} \qquad \therefore \frac{T_{n+2}}{T_{n+1}} = 1$

$\therefore \frac{(2n+1) - (n+1) + 1}{n+1} \times \frac{2b}{a} = 1$

$\frac{n+1}{n+1} \times \frac{2b}{a} = 1 \qquad \therefore a = 2b$

18 (b)

Solution :

$$A = \begin{pmatrix} 2 & 5 & 0 \\ 3 & 0 & -1 \\ 0 & 2 & -3 \end{pmatrix}$$

$\because |A| = 2(2) - 5(-9) = 49 \neq 0 \qquad \therefore \text{RK}(A) = 3$

$\therefore$ The system of equation has a unique solution which is the trivial solution.

19 (b)

Solution :

$z^2 = 1 + \omega^2 \qquad \therefore z^2 = -\omega$

$\therefore z = \pm\sqrt{-\omega} \qquad \therefore z = \pm\sqrt{i^2 \times \omega^4} = \pm\,\omega^2 i$

20 (a)

Solution :

$L \times M = (a\omega + b\omega^2)(a\omega^2 + b\omega)$

$= a^2\omega^3 + ab\omega^2 + ab\omega^4 + b^2\omega^3$

$= a^2 + ab(\omega^2 + \omega) + b^2 = a^2 - ab + b^2 \neq 1$

$\therefore$ L , M are not multiplicative inverse each of other.

21 (c)

22 (d)

Solution :

$$\text{length of perpendicular} = \frac{|2(3) + \sqrt{5}(0) + 4(-5) - 6|}{\sqrt{4 + 5 + 16}}$$

= 4 length unit.

23 (b)

Solution :

Direction vector of the straight line $(\overrightarrow{d}) = (1 , 5 , 4)$

$\therefore$ the points A (3 , 2 , 0) , B (3 , 6 , 4)

Both of them belongs to the plane.

$\because \overrightarrow{AB} = (0 , 4 , 4)$

$$\therefore \overrightarrow{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 5 & 4 \\ 0 & 4 & 4 \end{vmatrix} = 4\hat{i} - 4\hat{j} + 4\hat{k}$$

The equation of the plane is :

$(4 , -4 , 4) \cdot (x , y , z) = (4 , -4 , 4) \cdot (3 , 2 , 0)$

$\therefore 4x - 4y + 4z = 12 - 8$

$\therefore x - y + z - 1 = 0$

24 (c)

Solution :

let $\overline{AB} \cap$ yz − plane = {(0 , y , z)}

$\therefore \frac{2m_2 + 3m_1}{m_2 + m_1} = 0 \qquad \therefore 2m_2 + 3m_1 = 0$

$\therefore 3m_1 = -2m_2 \qquad \frac{m_1}{m_2} = \frac{-2}{3}$

$\therefore$ The plane divides $\overline{AB}$ by the ratio 2 : 3 externally

25 (c)

Solution :

$\because \lfloor 1 + \log x \rfloor = 1 \qquad \therefore 1 + \log x = 0$

and hence $\log x = -1$

$\therefore x = (10)^{-1} = \frac{1}{10}$ or $1 + \log x = 1$

and hence $\log x = 0 \qquad \therefore x = 1$

$\therefore$ S.S. $= \left\{\frac{1}{10} , 1\right\}$

Exam 17

1 (d)

Solution :

$({}^{10}C_x + {}^{10}C_{x+1}) + ({}^{10}C_{x+1} + {}^{10}C_{x+2}) = {}^{12}C_7$

$\therefore {}^{11}C_{x+1} + {}^{11}C_{x+2} = {}^{12}C_7$

$\therefore {}^{12}C_{x+2} = {}^{12}C_7$

$\therefore x + 2 = 7 \qquad \therefore x = 5$

or $x + 2 + 7 = 12 \qquad \therefore x = 3$

$\therefore$ Sum of possible values of $x = 5 + 3 = 8$

2 (b)

Solution :

$$\because \vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -2 \\ 1 & -3 & 4 \end{vmatrix}$$

$$= -2\hat{i} - 14\hat{j} - 10\hat{k}$$

$\therefore \| \vec{A} \times \vec{B} \| = \sqrt{4 + 196 + 100} = 10\sqrt{3}$

$\therefore$ Area of parallelogram $= 5\sqrt{3}$ square unit.

3 (d)

Solution :

Put $x = 0$ we get $\Delta = 0$

$$\therefore \begin{vmatrix} 1 & 3 & 1 \\ k & 2 & 3 \\ 0 & 2 & 1 \end{vmatrix} = 0 \ (R_2 - k R_1)$$

$$\therefore \begin{vmatrix} 1 & 3 & 1 \\ 0 & 2-3k & 3-k \\ 0 & 2 & 1 \end{vmatrix} = 0$$

$\therefore (2 - 3k) - 2(3 - k) = 0 \qquad \therefore k = -4$

4 (a)

Solution :

Matrix equation :

$$\begin{pmatrix} 0 & 3 & -1 \\ 2 & 1 & 1 \\ 1 & 2 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5 \\ 9 \\ 3 \end{pmatrix}$$

$$\therefore A = \begin{pmatrix} 0 & 3 & -1 \\ 2 & 1 & 1 \\ 1 & 2 & 2 \end{pmatrix}$$

$|A| = -12 \qquad \therefore RK(A) = 3$

$\therefore RK(A) = RK(A^*) =$ number of variables.

$\therefore$ The equations have a unique solution.

5 (b)

Solution :

$\vec{d}$ (direction vector of the straight line) $= (1, -1, 1)$

$\vec{n}$ (normal vector of the plane) $= (2, -1, 1)$

$$\because \cos\theta = \frac{|(1, -1, 1) \cdot (2, -1, 1)|}{\sqrt{1+1+1} \times \sqrt{4+1+1}}$$

$$= \frac{4}{\sqrt{3} \times \sqrt{6}} = \frac{2\sqrt{2}}{3}$$

Let $\propto$ is measure of angle between the plane and the straight line

$\therefore \theta + \propto = 90 \qquad \therefore \sin\propto = \cos\theta = \frac{2\sqrt{2}}{3}$

$\propto = \sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)$

6 (c)

Solution :

$$A = \begin{pmatrix} 3 & -2 & 1 \\ 6 & -5 & 2 \\ 9 & 6 & k \end{pmatrix}$$

$\therefore |A| = 3(-5k + 12) + 2(6k - 18) + 1(-36 + 45)$

$= -15k + 36 + 12k - 36 - 36 + 45 = -3k + 9$

$\therefore k = 3$

7 (b)

Solution :

$\vec{a} + \vec{b} + \vec{c} = \vec{O} \qquad \therefore \vec{a} + \vec{b} = -\vec{c} \qquad (1)$

Multiplying (1) : $(\times \vec{b})$

$\vec{a} \times \vec{b} + \vec{b} \times \vec{b} = -\vec{c} \times \vec{b}$

$\therefore \vec{a} \times \vec{b} = \vec{b} \times \vec{c}$

Multiplying (1) : $(\vec{a} \times)$

$\therefore \vec{a} \times \vec{a} + \vec{a} \times \vec{b} = -(\vec{a} \times \vec{c})$

$\therefore \vec{a} \times \vec{b} = \vec{c} \times \vec{a}$

$\therefore \vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$

8 (b)

Solution :

$\overrightarrow{AB} = (2, 1, -3) - (4, 2, -1) = (-2, -1, -2)$

$$\vec{F} = 10\sqrt{3} \times \frac{(1, -1, -1)}{\sqrt{1+1+1}} = (10, -10, -10)$$

$\therefore$ Work done $= \vec{F} \cdot \overrightarrow{AB}$

$= (-2, -1, -2) \cdot (10, -10, -10)$

$= -20 + 10 + 20 = 10$ work unit.

9 (c)

Solution :

$$T_{r+1} = \frac{1}{x^2} \times {}^{10}C_r \left(\frac{1}{x^3}\right)^r x^{10-r}$$

$$= {}^{10}C_r\, x^{-2} \times x^{-3r} \times x^{10-r} = {}^{10}C_r\, x^{8-4r}$$

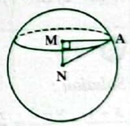
To find the free term of x

Put $8 - 4r = 0 \qquad \therefore r = 2$

$\therefore$ Free term is : $T_3 = {}^{10}C_2 = 45$

10 (b)

Solution :



Centre of the sphere is N (0, 0, 0)

and length of its radius

= 7 length unit

, MN = length of perpendicular segment.

From N to the plane $= \frac{|\text{zero} - 5\sqrt{14}|}{\sqrt{4+9+1}} = 5$ length unit.

$\therefore$ r of the circle $= \sqrt{7^2 - 5^2} = 2\sqrt{6}$ length unit.

$\therefore$ Area of the circle $= \pi r^2 = \pi\,(2\sqrt{6})^2$

$= 24\,\pi$ square unit.

11 (d)

Solution :

The equation of the straight line is

$$\frac{x-3}{1} = \frac{y-1}{-2} = \frac{z-3}{2}$$

$\because$ A (2, 3, 1) satisfies the equation of L

$\therefore A \in L$

$\therefore$ Length of perpendicular drawn from A to the straight line L = zero

12 (a)

Solution :

$\because z = \sin 20° + i\cos 20° = \cos 70° + i \sin 20°$

$\therefore \bar{z} = \cos 70° - i\sin 70° = \cos(-70°) + i\sin(-70°)$

$\therefore (\bar{z})^9 = \cos(-630°) + i\sin(-630°)$

$= \cos 90° + i\sin 90°$

sum of the amplitudes of cubic roots of the number $(\bar{z})^9$

$=$ Amplitudes of the number $(\bar{z})^9 = \frac{\pi}{2}$

13 (d)

Solution :

$\|\vec{A} - \vec{B}\|^2 = (\vec{A} - \vec{B}) \cdot (\vec{A} - \vec{B})$

$= \|\vec{A}\|^2 + \|\vec{B}\|^2 - 2(\vec{A} - \vec{B})$

$= \|\vec{A}\|^2 + \|\vec{B}\|^2 - 2\|\vec{A}\|\|\vec{B}\|\cos\theta$

$= 1 + 1 - 2 \times 1 \times 1\cos\theta$

$= 2 - 2\left(1 - 2\sin^2\frac{\theta}{2}\right) = 2 - 2 + 4\sin^2\frac{\theta}{2}$

$\therefore \|\vec{A} - \vec{B}\| = 2\sin\frac{\theta}{2}$

14 (c)

Solution :

$$\begin{pmatrix} x \\ y \end{pmatrix} = A^{-1}\begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 & 5 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 12 \\ 7 \end{pmatrix}$$

15 (a)

Solution :

$(x + y)^{100} - (x - y)^{100} = 2(T_2 + T_4 + T_6 + \dots + T_{100})$

$\therefore$ Number of terms = Number of the even order terms

= 50 terms.

16 (b)

Solution :

$(1 + \omega)^4 + (1 + \omega^2)^4 + (\omega + \omega^2)^4$

$= (-\omega^2)^4 + (-\omega)^4 + (-1)^4$

$= \omega^8 + \omega^4 + 1 = \omega^2 + \omega + 1 =$ zero

17 (d)

Solution :

$$M = 2 \times 3 \times 5 \times \begin{vmatrix} 1 & 0 & 3 \\ 2 & 3 & 5 \\ 1 & 4 & 2 \end{vmatrix} = 30\,N$$

18 (b)

Solution :

Number of ways : $^5C_3 = 10$

19 (a)

Solution :

$\overrightarrow{d_1} = (a, 2, 1)$, $\overrightarrow{d_2} = (2, 1, -1)$

$\cos\frac{\pi}{3} = \frac{|(a, 2, 1).(2, 1, -1)|}{\sqrt{a^2+4+1}\times\sqrt{4+1+1}}$

$\therefore \frac{1}{2} = \frac{|2a+2-1|}{\sqrt{a^2+5}\times\sqrt{6}}$

$\therefore 2|2a+1| = \sqrt{a^2+5}\times\sqrt{6}$

$\therefore 4(4a^2+4a+1) = 6a^2+30$

$\therefore 10a^2+16a-26=0$

$\therefore 5a^2+8a-13=0$ $\therefore (5a+13)(a-1)=0$

$\therefore a = \frac{-13}{5}$ or $a = 1$

20 (b)

Solution :

$|Z| = |Z+\bar{Z}|$ $\therefore \sqrt{x^2+3} = |2x|$ by squaring

$x^2+3 = 4x^2$ $\therefore 3x^2 = 3$

$\therefore x^2 = 1$ $\therefore x = \pm 1$

21 (c)

Solution :

$\left(\frac{\bar{z}}{z}\right)^3 = \left(\frac{1+\omega^2}{1+\omega}\right)^3 = \left(\frac{-\omega}{-\omega^2}\right)^3 = \left(\frac{1}{\omega}\right)^3 = 1$

22 (b)

Solution :

direction vector of the straight line is

$\vec{d} = \left(1, \frac{1}{3}, \frac{1}{k}\right)$

the normal vector of the plane is $\vec{n} = (1, 3, 2)$

$\because$ The straight line // the plane

$\therefore \vec{d} \perp \vec{n}$ $\therefore \vec{d}.\vec{n} = 0$

$\therefore \left(1, \frac{1}{3}, \frac{1}{k}\right).(1, 3, 2) = 0$

$\therefore 1+1+\frac{2}{k} = 0$ $\therefore k = -1$

23 (b)

Solution :

$\frac{T_{r+1}}{T_r} = \frac{1}{4}$, $\frac{n-r+1}{r} = \frac{1}{4}$

$\therefore \frac{24-r+1}{r} = \frac{1}{4}$ $\therefore 100-4r = r$

$\therefore 5r = 100$, $r = 20$

$\therefore$ The two terms are T_{20} , T_{21}

24 (c)

Solution :

$\because$ $^{n+2}C_3$, $^nP_2$, $^nP_2$ in geometrical seqent

$\therefore (^nP_2)^2 = {}^{n+2}C_3 \times {}^nC_2$

$\therefore (n(n-1))^2 = \frac{(n+2)(n+1)(n)}{6} \times \frac{n(n-1)}{2}$

$\therefore (n-1) = \frac{(n+2)(n+1)}{12}$

$\therefore 12n-12 = n^2+3n+2$ $\therefore n^2-9n+14=0$

$\therefore (n-2)(n-7)=0$ $\therefore n = 2$ or $n = 7$

25 (c)

Solution :

$\frac{x+1}{2} = \frac{y-2}{1} = \frac{z}{3} = k$

$\therefore x = 2t-1$, $y = t+2$, $z = 3t$

$(2t-1)-2(t+2)+3(3t)+5=0$

$9t = 0$ $\therefore t = 0$

$\therefore$ The point is $(-1, 2, 0)$

Exam 18

1 (c)

Solution :

$\left(1+\frac{1}{\omega}+i\right)\left(1+\frac{1}{\omega^2}+i\right) = (1+\omega^2+i)(1+\omega+i)$

$= (-\omega+i)(-\omega^2+i)$

$= \omega^3 - i\omega^2 - i\omega + i^2$

$= 1 - i(\omega^2+\omega) - 1$

$= -i(-1) = i$

2 (c)

Solution :

$\because (1, -1, 2) \in$ the straight line.

$\therefore$ The vector $(1, -1, 2) - (2, -1, 3)$

$= (-1, 0, -1) \in$ required plane.

$\because (2, 2, -1)$ is direction vector of the straight line.

$\therefore$ Normal direction vector of the plane

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 0 & -1 \\ 2 & 2 & -1 \end{vmatrix} = 2\hat{i} - 3\hat{j} - 2\hat{k}$$

$\therefore$ Equation of the required plane.

$\vec{r}.(2, -3, -2) = (1, -1, 2).(2, -3, -2)$

$2x - 3y - 2z - 1 = 0$

3 (b)

Solution :

$$\Delta = \begin{vmatrix} 2x & 2y & 2z \\ 5l+x & 5m+y & 5n+z \\ 7k-3l & 7p-3m & 7q-3n \end{vmatrix}$$

$$= 2\begin{vmatrix} x & y & z \\ 5l+x & 5m+y & 5n+z \\ 7k-3l & 7p-3m & 7q-3n \end{vmatrix}$$

by doing $(R_2 - R_1)$

$$= 2\begin{vmatrix} x & y & z \\ 5l & 5m & 5n \\ 7k-3l & 7p-3m & 7q-3n \end{vmatrix}$$

by taking 5 a common factor from R_2

$$= 10\begin{vmatrix} x & y & z \\ l & m & n \\ 7k-3l & 7p-3m & 7q-3n \end{vmatrix}$$

by doing $(R_3 + 3R_2)$

$$= 10\begin{vmatrix} x & y & z \\ l & m & n \\ 7k & 7p & 7q \end{vmatrix} = 70\begin{vmatrix} x & y & z \\ l & m & n \\ k & p & q \end{vmatrix}$$

$= 70 \times 2 = 140$

4 (b)

Solution :

The equation of the sphere is

$2x^2+2y^2+2z^2-2x+4y-6z-1=0$

$\therefore x^2+y^2+z^2-x+2y-3z-\frac{1}{2}=0$

$\therefore r = \sqrt{\left(\frac{1}{2}\right)^2+(-1)^2+\left(\frac{3}{2}\right)^2+\frac{1}{2}} = 2$ length unit.

$\therefore$ the equation of the required sphere

$(x-1)^2+(y+1)^2+(z-1)^2=4$

$x^2+y^2+z^2-2x+2y-2z-1=0$

5 (c)

Solution :

$\overrightarrow{d_1} = (2, 1, 1)$, $\overrightarrow{d_2} = (a, 2, b)$

$\because$ the two straight lines are parallel

$\therefore \frac{a}{2} = \frac{2}{1} = \frac{b}{1}$

$\therefore a = 4$, $b = 2$, $a+b=6$

6 (a)

Solution :

Matrix equation : $\begin{pmatrix} 1 & 2 & 1 \\ 4 & -1 & -1 \\ 1 & 1 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 6 \\ 10 \end{pmatrix}$

$A = \begin{pmatrix} 1 & 2 & 1 \\ 4 & -1 & -1 \\ 1 & 1 & 3 \end{pmatrix}$ $\therefore |A| = -23 \neq$ zero

RK (A) = RK (A*) = 3 = Number of variables.

$\therefore$ The equations have unique solution.

7 (b)

Solution :

$A = \begin{pmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -5 & 0 & 1 \end{pmatrix}$ $\therefore |A| = 1$

$\text{Adj}(A) = \begin{pmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 5 & 0 & 1 \end{pmatrix}$

$\therefore A^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 5 & 0 & 1 \end{pmatrix}$

8 (b)

Solution :

the direction angles of the vector is $(30°, 90°, 60°)$

$\therefore$ Direction cosines $= (\cos 30°, \cos 90°, \cos 60°)$

$$= \left(\frac{\sqrt{3}}{2}, 0, \frac{1}{2}\right)$$

9 (c)

Solution :

$\because \overrightarrow{A} \parallel \overrightarrow{B}$ $\qquad \therefore \frac{4}{2} = \frac{-k}{2} = \frac{6}{m}$

$\therefore k = -4 \quad , \quad m = 3$

$\therefore k + m = -4 + 3 = -1$

10 (b)

11 (d)

12 (b)

Solution :

$\frac{T_{r+1}}{T_r} \geq 1$ $\qquad \therefore \frac{6 - r + 1}{r} \times \frac{2}{3} \geq 1$

$\frac{14 - 2r}{3r} \geq 1$ $\qquad \therefore 14 - 2r \geq 3r$

$\therefore r \leq \frac{14}{5}$

$\frac{T_{r+2}}{T_{r+1}} \leq 1$ $\qquad \therefore \frac{6 - (r+1) + 1}{r + 1} \times \frac{2}{3} \leq 1$

$\therefore \frac{12 - 2r}{3r + 3} \leq 1$ $\qquad \therefore 12 - 2r \leq 3r + 3$

$\therefore r \geq \frac{9}{5}$ $\qquad \therefore \frac{9}{5} \leq r \leq \frac{14}{5}$

$\because r \in N$ $\qquad \therefore r = 2$

$\therefore T_3$ is the term which has greatest cofficient in the expansion

13 (d)

Solution :

$\overrightarrow{N_1} = (1, 2, k)$, $\overrightarrow{n_2} = (2, 1, -2)$

$\therefore \overrightarrow{n_1} \cdot \overrightarrow{n_2} = 0$

$\therefore (1, 2, k) \cdot (2, 1, -2) = 0$

$\therefore 2 + 2 - 2k = 0 \qquad k = 2$

14 (c)

Solution :

$$z = \frac{1 + i \times \frac{\sin 18}{\cos 18}}{1 - i \times \frac{\sin 18}{\cos 18}} = \frac{\cos 18 + i \sin 18}{\cos 18 - i \sin 18}$$

$$= \frac{\cos 18 + i \sin 18}{\cos(-18) + i \sin(-18)}$$

Amplitude of $z = 18 - (-18) = 36$

$\therefore$ Amplitude of $z = \frac{1}{5}\pi$

15 (c)

Solution :

$$\frac{\lfloor\underline{n}}{\lfloor\underline{3}\,\lfloor\underline{n-3}} \times \frac{\lfloor\underline{4}\,\lfloor\underline{n-5}}{\lfloor\underline{n-1}} = \frac{8}{5}$$

$$\frac{n\,\lfloor\underline{n-1}}{\lfloor\underline{3}\,(n-3)(n-4)\,\lfloor\underline{n-5}} \times \frac{4\,\lfloor\underline{3}\,\lfloor\underline{n-5}}{\lfloor\underline{n-1}} = \frac{8}{5}$$

$\therefore \frac{n}{n^2 - 7n + 12} = \frac{2}{5}$

$\therefore 2n^2 - 14n + 24 - 5n = 0$

$\therefore 2n^2 - 19n + 24 = 0$

$\therefore (2n - 3)(n - 8) = 0$

$\therefore n = \frac{2}{3}$ (refused) or $n = 8$

16 (c)

Solution :

Put $\frac{x - 5}{1} = \frac{y - 3}{3} = \frac{z - 1}{-2} = t$

$\therefore x = 5 + t \quad , \quad y = 3 + 3t \quad , \quad z = 1 - 2t$

By substitute in equation of the sphere

$\therefore (5 + t)^2 + (3 + 3t)^2 + (1 - 2t)^2 - 2(5 + t) - 4(3 + 3t)$

$-2(1 - 2t) - 39 = 0$

$\therefore t^2 + t - 2 = 0$ $\qquad \therefore (t + 2)(t - 1) = 0$

$t = -2$ or $t = 1$

$\therefore$ the intersection points are :

$(5 - 2, 3 - 6, 1 + 4) = (3, -3, 5)$

or $(5 + 1, 3 + 3, 1 - 2) = (6, 6, -1)$

$\therefore$ Length of intercepted part by the sphere is the distant between the two points $= \sqrt{3^2 + 9^2 + (-6)^2}$

$= \sqrt{126}$ unit length.

17 (b)

Solution :

$\because$ The direction vector $\overrightarrow{d_1} = (9, 4, -2)$

, the normal direction vector of the plane is $d_2 = (3, 4, 5)$

$\because \cos\theta = \frac{|(9, 4, -2) \cdot (3, 4, 5)|}{\sqrt{81 + 16 + 4}\sqrt{9 + 16 + 25}} = \frac{33}{\sqrt{101}\sqrt{50}}$

$\therefore$ The measure of the angle $= 90° - \cos^{-1}\frac{33}{\sqrt{5050}}$

$\simeq 90° - 62°\,20' = 27°\,40'$

18 (c)

Solution :

$\because T_{10} = T_9$ $\qquad \therefore \frac{T_{10}}{T_9} = 1$

$\therefore \frac{20 - 9 + 1}{9} \times \frac{3}{2x^3} = 1$

$x^3 = 2$ $\qquad \therefore x = \sqrt[3]{2}$

19 (b)

Solution :

$\overrightarrow{CD} \cdot \overrightarrow{CB} = (\overrightarrow{CB} + \overrightarrow{BD}) \cdot \overrightarrow{CB}$

$= \overrightarrow{CB} \cdot \overrightarrow{CB} + \overrightarrow{BD} \cdot \overrightarrow{CB}$

$= \|\overrightarrow{CB}\|^2 - \overrightarrow{BD} \cdot \overrightarrow{BC}$

$= \|\overrightarrow{CB}\|^2 - \|\overrightarrow{BD}\|\,\|\overrightarrow{BC}\| \cos 60°$

$= 36 - 2 \times 6 \times \frac{1}{2} = 30$

20 (d)

Solution :

$$\overrightarrow{A} \cdot \overrightarrow{B} \times \overrightarrow{C} = \begin{vmatrix} 3 & -4 & 5 \\ 0 & -4 & 3 \\ 0 & 0 & 5 \end{vmatrix} = -60$$

$\therefore$ Volume of the parallelpiped $= |-60|$

$= 60$ cubic unit

21 (b)

Solution :

$\|\overrightarrow{A}\| \cos\theta = 3$

$\therefore 6\cos\theta = 3$ $\qquad \therefore \cos\theta = \frac{1}{2}$

Component of $\overrightarrow{B}$ in direction $\overrightarrow{A}$

$= \|\overrightarrow{B}\| \cos\theta = 4 \times \frac{1}{2} = 2$

22 (c)

Solution :

$$A^* = \left(\begin{array}{cc:c} 2 & -3 & 5 \\ 6 & 1 & 15 \end{array}\right)$$

$\because \begin{vmatrix} 2 & -3 \\ 6 & 1 \end{vmatrix} = 20 \neq 0$ $\qquad \therefore RK(A^*) = 2$

23 (d)

Solution :

the ordered of the two middle terms

are $\frac{11 + 1}{2} = 6$, $\frac{11 + 3}{2} = 7$

$\therefore \frac{11 - 6 + 1}{6} \times \frac{-4}{x} \times \frac{4}{x^3} = -1$

$\therefore \frac{-16}{x^4} = -1$ $\qquad x^4 = 16$

$\therefore x = \pm 2$

24 (c)

Solution :

${}^n c_1 (-m x) = -\frac{1}{4}x$

$\therefore n m = \frac{1}{4}$ (1)

${}^n c_2 \times (-m x)^2 = \frac{3}{100}x^2$

$\therefore \frac{n(n-1)}{2} \times m^2 x^2 = \frac{3}{100}x^2$

$\therefore n m (n m - m) = \frac{3}{50}$ (2)

by substitute (1) in (2) : $\therefore \frac{1}{4}\left(\frac{1}{4} - m\right) = \frac{3}{50}$

$\therefore m = \frac{1}{100}$

25 (d)

Solution :

$z = \left(\cos\left(\frac{-\pi}{6}\right) + i \sin\left(\frac{-\pi}{6}\right)\right)\left(\cos\left(\frac{\pi}{4}\right) + i \sin\left(\frac{\pi}{4}\right)\right)$

$= \left(\cos\left(\frac{\pi}{12}\right) + i \sin\left(\frac{\pi}{12}\right)\right)$

$z = e^{\frac{\pi}{12}i}$

Exam 19

1 (d)

Solution :

$\because \begin{vmatrix} 4 & -2 \\ 10 & 5 \end{vmatrix} = 40 \neq 0$

$\therefore$ The matrix $\begin{pmatrix} 4 & -2 \\ 10 & 5 \end{pmatrix}$ has multiplicative inverse.

2 (d)

Solution :

$\because {}^nc_r = {}^nP_r$ $\qquad \therefore \dfrac{{}^nP_r}{\underline{|r}} = {}^nP_r$

$\therefore \underline{|r} = 1$ $\qquad \therefore r = 0$ or 1

3 (a)

Solution :

order of the middle term $= \dfrac{2n+2}{2} = n+1$

coefficient of $T_{n+1} = {}^{2n}c_n = \dfrac{\underline{|2n}}{\underline{|n}\,\underline{|n}}$

$= \dfrac{2n(2n-1)(2n-2)\times \dots \times 4\times 3\times 2\times 1}{\underline{|n}\,\underline{|n}}$

$= \dfrac{(2\times 4\times 6\times \dots \times 2n)\times (1\times 3\times 5\times \dots \times (2n-1))}{\underline{|n}\,\underline{|n}}$

$= \dfrac{2^n \cancel{\underline{|n}}(1\times 3\times 5\times \dots \times (2n-1))}{\underline{|n}\,\cancel{\underline{|n}}}$

$= \dfrac{1\times 3\times 5\times \dots \times (2n-1)\times 2^n}{\underline{|n}}$

4 (c)

Solution :

$\because \vec{d_1} = (2, -1, 1), \vec{d_2} = (-2, 7, 11)$

$\because \vec{d_1} \cdot \vec{d_2} = (2, -1, 1) \cdot (-2, 7, 11)$

$= -4 - 7 + 11 = 0 = L_1 \perp L_2$

at the point of intersection $\vec{r_1} = \vec{r_2}$

$\therefore (1, 2, 4) + t_1 (2, -1, 1)$

$= (1, 1, 1) + t_2 (-2, 7, 11)$

$\therefore 1 + 2t_1 = 1 - 2t_2$ $\quad$ then $t_1 + t_2 = 0$ $\quad$ (1)

$, 2 - t_1 = 1 + 7t_2$ $\quad$ then $t_1 + 7t_2 = 1$ $\quad$ (2)

$, 4 + t_1 = 1 + 11t_2$ $\quad$ then $t_1 - 11t_2 = -3$ $\quad$ (3)

from (1) , (2) : $t_1 = -\frac{1}{6}$, $t_2 = \frac{1}{6}$

By substituting in (3)

$t_1 - 11t_2 = \left(\frac{-1}{6}\right) - 11\left(\frac{1}{6}\right) = -2 \neq -3$

$\therefore$ They have no intersection points

then the two straight lines are skew and perpendicular

5 (c)

Solution :

By taking 4 a common factor from R_2 in each determinant

$\therefore$ determinant $= 4\begin{vmatrix} 3 & 5 & 6 \\ 2 & -1 & 3 \\ 4 & 7 & 5 \end{vmatrix}$

$-4\begin{vmatrix} -2 & 1 & -3 \\ 3 & 5 & 6 \\ 4 & 7 & 5 \end{vmatrix}$

by interchange R_1 , R_2 in the 2^{nd} det.

$\therefore$ determinant $= 4\begin{vmatrix} 3 & 5 & 6 \\ 2 & -1 & 3 \\ 4 & 7 & 5 \end{vmatrix}$

$+4\begin{vmatrix} 3 & 5 & 6 \\ -2 & 1 & -3 \\ 4 & 7 & 5 \end{vmatrix} = 4\begin{vmatrix} 3 & 5 & 6 \\ 0 & 0 & 0 \\ 4 & 7 & 5 \end{vmatrix} =$ zero

6 (a)

Solution :

$A = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ 1 & -1 & 0 \end{pmatrix}$ $\quad \therefore |A| = -1 \neq$ zero

$\because$ RK (A) = RK (A*) = 3 = number of variables.

$\therefore$ The equations have unique solution.

7 (c)

Solution :

$\vec{A} + \vec{B} + \vec{C} = \vec{O}$ $\qquad \therefore \vec{A} + \vec{B} = -\vec{C}$

$\therefore (\vec{A} + \vec{B}) \cdot (\vec{A} + \vec{B}) = -\vec{C} \cdot -\vec{C}$

$\therefore \|\vec{A}\|^2 + \|\vec{B}\|^2 + 2(\vec{A} \cdot \vec{B}) = \|\vec{C}\|^2$

$\therefore 36 + 64 + 2(\vec{A} \cdot \vec{B}) = 100$ $\qquad \therefore \vec{A} \cdot \vec{B} = 0$

$\therefore \vec{A} \cdot \vec{B} + \vec{B} \cdot \vec{C} + \vec{C} \cdot \vec{A} = \text{zero} + \vec{C} \cdot (\vec{A} + \vec{B})$

$= \vec{C} \cdot (-\vec{C}) = -\|\vec{C}\|^2$

$= -100$

8 (c)

Solution :

$\because \|\vec{r}\|^2 - \vec{r} \cdot (2, 4, -2) = 10$

$\therefore x^2 + y^2 + z^2 - (x, y, z) \cdot (2, 4, -2) = 10$

$\therefore x^2 + y^2 + z^2 - 2x - 4y + 2z - 10 = 0$

$\therefore$ the equation represents equation of sphere its radius $r = \sqrt{1^2 + 2^2 + (-1)^2 + 10} = 4$ length unit.

9 (c)

Solution :

Amp. $(z_1 z_2^3)$ = Amp. (z_1) + 3 × Amp. (z_2) = 81 $\quad$ (1)

Amp. $\left(\frac{z_1}{z_2}\right)$ = Amp. (z_1) − Amp. (z_2) = 33 $\quad$ (2)

by solving the two equations :

Amp. $(z_1) = 45°$, Amp. $(z_2) = 12°$

$\therefore z_1 = \cos 45° + i \sin 45°$, $z_2 = \cos 12° + i \sin 12°$

$z_1^{15} z_2^{15} = (z_1 z_2)^{15} = (\cos 57° + i \sin 57°)^{15}$

$= \cos 135° + i \sin 135° = \frac{-1}{\sqrt{2}} + \frac{1}{\sqrt{2}} i$

10 (d)

Solution :

$\cos\theta = \dfrac{|(3, -1, 1) \cdot (1, 4, -2)|}{\sqrt{9+1+1}\sqrt{1+16+4}}$

$\therefore \theta = \cos^{-1}\left(\dfrac{3}{\sqrt{11}\sqrt{21}}\right) \approx 78° \; 37'$

11 (b)

Solution :

$\overrightarrow{BC} = (2, 4, -3) - (-2, 1, 2) = (4, 3, -5)$

$\therefore$ direction vector of the straight line.

$= \overrightarrow{BC} = (4, 3, -5)$

$\because C \in$ the straight line $\overleftrightarrow{BC}$

$\therefore$ the equation of the straight line is :

$\vec{r} = (2, 4, -3) + t(4, 3, -5)$

12 (b)

Solution :

$\because T_{10} = T_9$ $\qquad \therefore \dfrac{T_{10}}{T_9} = 1$

$\therefore \dfrac{n-9+1}{9}\left(\dfrac{3}{2x^3}\right) = 1$ $\qquad \therefore (n-8) = 6x^3$ $\quad$ (1)

$T_6 : T_7 = 8 : 15$ $\qquad \therefore \dfrac{T_7}{T_6} = \dfrac{15}{8}$

$\therefore \dfrac{n-6+1}{6}\left(\dfrac{3}{2x^3}\right) = \dfrac{15}{8}$ $\qquad \therefore 2(n-5) = 15x^3$ (2)

By dividing (1) , (2) : $\therefore \dfrac{2(n-5)}{(n-8)} = \dfrac{15x^3}{6x^3}$

$4(n-5) = 5(n-8)$ $\qquad \therefore n = 20$

13 (d)

Solution :

${}^{n+2}P_r = 2\,{}^{n+2}C_r$ $\qquad \therefore \dfrac{\underline{|n+2}}{\underline{|n+2-r}} = 2\dfrac{\underline{|n+2}}{\underline{|n+2-r}\,\underline{|r}}$

$\underline{|r} = 2 \times 1$ $\qquad \therefore r = 2$

$\dfrac{{}^nC_{r+1}}{{}^nC_r} = \dfrac{5}{3}$ $\qquad \therefore \dfrac{{}^nC_3}{{}^nC_2} = \dfrac{5}{3}$

$\dfrac{n-3+1}{3} = \dfrac{5}{3}$ $\qquad \therefore n = 7$

$\therefore {}^{2n}C_{n-r} + {}^{n+3}P_{r-1} = {}^{14}C_5 + {}^{10}P_1 = 2012$

14 (c)

Solution :

$\vec{d_1} = (2, 0, -2)$, $\vec{d_2} = (1, 2, -2)$

$\cos\theta = \dfrac{|(2, 0, -2) \cdot (1, 2, -2)|}{\sqrt{4+0+4} \times \sqrt{1+4+4}} = \dfrac{6}{6\sqrt{2}} = \dfrac{1}{\sqrt{2}}$

$\therefore \theta = 45°$

15 (c)

Solution :

$(a + b\omega + a\omega^2)(a + b\omega^2 + a\omega)$

$= (a(1+\omega^2) + b\omega)(a(1+\omega) + b\omega^2)$

$= (-a\omega + b\omega)(-a\omega^2 + b\omega^2)$

$= (b-a)\omega \times (b-a)\omega^2 = (b-a)^2\omega^3$

$= (b-a)^2 = (a-b)^2$

16 (a)

Solution :

$r = \sqrt{3^2 + 4^2 + 5^2 - 1} = 7$ length unit.

17 (a)

Solution :

$$\begin{vmatrix} ab & a & \frac{1}{c} \\ ac & c & \frac{1}{b} \\ bc & b & \frac{1}{a} \end{vmatrix} = \frac{1}{a} \times \frac{1}{b} \times \frac{1}{c} \begin{vmatrix} abc & ac & 1 \\ abc & bc & 1 \\ abc & ab & 1 \end{vmatrix}$$

$$= abc \times \frac{1}{abc} \times \begin{vmatrix} 1 & ac & 1 \\ 1 & bc & 1 \\ 1 & ab & 1 \end{vmatrix}$$

$= 1 \times 0 = 0$

18 (b)

Solution :

$\overrightarrow{A} = (-2 , 1 , 2)$ $\quad \therefore \|\overrightarrow{A}\| = \sqrt{4+1+4} = 3$

$\therefore$ direction cosines of vector $\overrightarrow{A} = \left(\frac{-2}{3} , \frac{1}{3} , \frac{2}{3}\right)$

19 (b)

Solution :

$\overrightarrow{A} = (1 , 1 , 1) , \overrightarrow{B} = (4 , 5 , 1) , \overrightarrow{C} = (5 , -2 , 1)$

$\overrightarrow{AB} = (3 , 4 , 0) , \overrightarrow{AC} = (4 , -3 , 0)$

$$\therefore \overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 0 \\ 4 & -3 & 0 \end{vmatrix} = -25\hat{k}$$

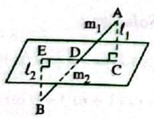
$\therefore \|\overrightarrow{AB} \times \overrightarrow{AC}\| = 25$

$\therefore$ The area of the Δ ABC $= \frac{1}{2} \|\overrightarrow{AB} \times \overrightarrow{AC}\|$

$= \frac{25}{2}$ square unit.

20 (c)

Solution :



$\because 2(1) + 3(1) - (1) - 5 = -1 < 0$

$\because 2(3) + 3(2) + 1 - 5 = 8 > 0$

$\therefore$ A , B in different sides from the plane

$\therefore$ The plane divides $\overline{AB}$ internally by ratio $m_1 : m_2$

From the similarity of $\Delta\Delta$ ACD , BED

then $\frac{m_1}{m_2} = \frac{\ell_1}{\ell_2}$

where ℓ_1 , ℓ_2 are lengths of the two perpendicular drawn from A and B on the plane respectivly

$\because \ell_1 = \frac{|2(1) + 3(1) - (1) - 5|}{\sqrt{4+9+1}} = \frac{1}{\sqrt{14}}$

$\ell_2 = \frac{|2(3) + 3(2) - (-1) - 5|}{\sqrt{4+9+1}} = \frac{8}{\sqrt{14}}$

$\ell_1 : \ell_2 = 1 : 8 \quad , \quad m_1 : m_2 = 1 : 8$

21 (d)

Solution :

the order of the middle term $= \frac{8+2}{2} = 5$

$\therefore T_5$ is the middle term

$\because T_{r+1} = {}^8c_r \left(\frac{1}{a\,x}\right)^r (x^2)^{8-r} = {}^8c_r \left(\frac{1}{a}\right)^r \times x^{16-3r}$

put $16 - 3r = 7$ $\quad \therefore 3r = 9 \quad \therefore r = 3$

$\therefore T_4$ is the term containing x^7

$\because$ coefficient of T_5 = coefficient of T_4

$\therefore \frac{\text{coefficient } T_5}{\text{coefficient } T_4} = 1 \quad \therefore \frac{8-4+1}{4} \times \frac{1}{a} = 1$

$\therefore \frac{5}{4} \times \frac{1}{a} = 1 \quad \therefore a = \frac{5}{4}$

22 (b)

Solution :

$\left(\frac{z_1}{z_2}\right) = \frac{r_1\left(\cos(100°) + i\sin(100°)\right)}{r_2\left(\cos(-170°) + i\sin(-170°)\right)}$

$= \frac{r_1}{r_2} (\cos 270° + i\sin 270°)$

$= \frac{r_1}{r_2} \left(\cos(-90°) + i\sin(-90°)\right)$

the principle amplitude of the number $\left(\frac{z_1}{z_2}\right) = -\frac{\pi}{2}$

23 (a)

Solution :

$x + y i = e^{\theta i} = \cos\theta + i\sin\theta$

$, \cos\theta = x \quad , \quad \sin\theta = y$

$\therefore x y = \sin\theta \cdot \cos\theta = \frac{1}{2}\sin 2\theta$

$\therefore x y \in \left[\frac{-1}{2} , \frac{1}{2}\right]$

24 (a)

Solution :

Value of the determinent $= \log_3 L + \log_3 M$

$= \log_3 (LM)$

$\because$ L , M are the two roots of the equation

$x^2 - 11x + 27 = 0$

$\therefore LM = 27$

$\therefore$ Value of the determinent $= \log_3 27 = 3$

25 (b)

Solution :

$\because 1 + 20x + 190x^2 + ... + x^{20} = (1+x)^{20}$

$, x^{18} + 18x^{17} + 153x^{16} + ... + 1 = (1+x)^{18}$

$\therefore (1+x)^{20} = (1+x)^{18}$

$\therefore (1+x)^{20} - (1+x)^{18} = 0$

$(1+x)^{18}\left[(1+x)^2 - 1\right] = 0$

$\therefore (1+x)^{18} = 0 \quad \therefore x = -1$

Or $(1+x)^2 - 1 = 0 \quad \therefore (1+x)^2 = 1$

$\therefore 1 + x = \pm 1 \quad \therefore 1 + x = 1$, then $x = 0$

Or $1 + x = -1$, then $x = -2$

$\therefore$ S.S. $= \{-1 , 0 , -2\}$

Exam 20

1 (c)

2 (a)

Solution :

$T_{r+1} = {}^{3n}C_r \left(\frac{1}{2x}\right)^r (x^2)^{3n-r}$

$\therefore T_{r+1} = {}^{3n}C_r \times 2^{-r} \times x^{6n-3r}$

At the free term of x

Then : $6n - 3r = 0 \quad \therefore r = 2n$

$\therefore$ The free term of x is T_{2n+1}

At n = 4 , then the free term of x is T_9

$\therefore$ The middle term is T_7

$\therefore \frac{T_9}{T_7} = \frac{T_9}{T_8} \times \frac{T_8}{T_7} = \frac{12-8+1}{8} \times \frac{12-7+1}{7} \times \left(\frac{1}{2}\right)^2$

$= \frac{15}{112}$

3 (c)

Solution :

The normal direction vector of the required plane

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 3 & 3 & 1 \end{vmatrix} = -7\hat{i} + 8\hat{j} - 3\hat{k}$$

$\therefore$ The equation of the required plane is :

$\overrightarrow{r} \cdot (-7 , 8 , -3) = (-1 , 3 , 2) \cdot (-7 , 8 , -3)$

$\therefore -7x + 8y - 3z - 25 = 0$

4 (b)

5 (a)

Solution :

$\left(\omega^2 + \frac{1}{\omega}\right)\left(1 + \frac{1}{\omega^2}\right)^2 = (\omega^2 + \omega^2)(1 + \omega)^2$

$= 2\omega^2 \times (-\omega^2)^2 = 2\omega^2 \times \omega^4$

$= 2\omega^6 = 2 \times 1 = 2$

6 (a)

Solution :

number of diagonals of the polygon $= {}^nc_2 - n$

$\therefore {}^nc_2 - n = 44$

$\therefore \frac{1}{2} n(n-1) - n = 44$

$\therefore \frac{1}{2} n^2 - \frac{1}{2} n - n - 44 = 0$

$\therefore n^2 - 3n - 88 = 0$

$\therefore n = 11$ or $n = -8$ (refused)

$\therefore$ number of sides = 11 sides

7 (d)

8 (c)

Solution :

$\cos^2\theta_x + \cos^2 60° + \cos^2 60° = 1$

$\therefore \cos^2\theta_x + \frac{1}{4} + \frac{1}{4} = 1$

$\therefore \cos^2\theta_x = \frac{1}{2} \quad \therefore \cos\theta_x = \pm\frac{1}{\sqrt{2}}$

$\therefore \theta_x = 45°$ or $135°$

9 (c)

Solution :

$\because z = 1 + \sqrt{3}\,i \quad , \quad r = \sqrt{1+3} = 2$

$, x > 0 \quad , \quad y > 0 \quad \therefore \theta = \tan^{-1}\left(\frac{\sqrt{3}}{1}\right) = \frac{\pi}{3}$

$\therefore z = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right) = 2e^{\frac{\pi}{3}i}$

$\sqrt{z} = \sqrt{2}\,e^{\left(\frac{\frac{\pi}{3} + 2\pi k}{2}\right)i}$ where k = 0 , 1

at k = 0 $\quad \therefore \sqrt{z} = \sqrt{2}\,e^{\frac{\pi}{6}i}$

at k = 1 $\quad \therefore \sqrt{z} = \sqrt{2}\,e^{\frac{-5\pi}{6}i}$

10 (d)

Solution :

$$|A| = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 3 & 4 & k \end{vmatrix} = (2k-12)-(k-9)-2$$

$$= k-5$$

to get a unique solution

$\therefore |A| \neq 0$ $\therefore k \neq 5$

$\therefore k \in \mathbb{R} - \{5\}$

11 (b)

Solution :

$\underline{|n} = 720 = \underline{|6}$ $\therefore n = 6$

$\because {}^7P_{r-2} = 210 = 7 \times 6 \times 5 = {}^7P_3$

$\therefore r-2 = 3$ $\therefore r = 5$

$\therefore {}^{n+1}C_r + {}^{n+1}C_{r-1} = {}^{n+2}C_r = {}^8C_5 = 56$

12 (a)

Solution :

the point B (3 , 6 , 4) belongs to the straight line and the plane ,

$\overrightarrow{BA} = (3,2,0)-(3,6,4) = (0,-4,-4)$

$\vec{d}$ direction vector of the straight line. $= (1,5,4)$

, $\vec{n}$ (normal direction vector of the plane)

$$= \overrightarrow{BA} \times \vec{d} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & -4 & -4 \\ 1 & 5 & 4 \end{vmatrix} = 4\hat{i} - 4\hat{j} + 4\hat{k}$$

$\therefore$ the equation of the plane is

$(4,-4,4) \cdot (x,y,z) = (4,-4,4) \cdot (3,2,0)$

$4x - 4y + 4z = 12 - 8$

$\therefore 4x - 4y + 4z = 4$

$\therefore x - y + z = 1$

13 (c)

Solution :

Equation of the sphere is

$x^2 - x_1 x - x_2 x + x_1 x_2 + y^2 - y_1 y - y_2 y + y_1 y_2$

$+ z^2 - z_1 z - z_2 z + z_1 z_2 = 0$

$\therefore x^2 + y^2 + z^2 - (x_1 + x_2)x - (y_1 + y_2)y$

$-(z_1 + z_2)z + x_1 x_2 + y_1 y_2 + z_1 z_2 = 0$

then its centre is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$

14 (c)

Solution :

$\overrightarrow{AB} = (-3,2,-3)$, $\overrightarrow{CD} = (1,1,1)$

$\therefore$ projection of $\overrightarrow{AB}$ in direction of $\overrightarrow{CD}$

$$= \frac{\overrightarrow{AB} \cdot \overrightarrow{CD}}{\|\overrightarrow{CD}\|} = \frac{(-3,2,-3) \cdot (1,1,1)}{\sqrt{1+1+1}}$$

$$= \frac{-3+2-3}{\sqrt{3}} = \frac{-4}{\sqrt{3}}$$

15 (d)

Solution :

$$x \times x \times x \begin{vmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \\ 1 & -1 & 0 \end{vmatrix} = 96$$

$x^3 \times (-12) = 96$

$\therefore x^3 = -8$

$\therefore x = -2$ $\therefore$ S.S. $= \{-2\}$

16 (c)

Solution :

$\vec{n}_1 = (3,-1,1)$, $\vec{n}_2 = (1,1,-2)$

$$\vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 1 \\ 1 & 1 & -2 \end{vmatrix} = \hat{i} + 7\hat{j} + 4\hat{k}$$

$\therefore$ The intersection line of the two planes is parallel to the vector (1 , 7 , 4)

17 (c)

Solution :

$\because \vec{A} \cdot \vec{B} = \|\vec{A}\| \|\vec{B}\| \cos\theta$

$\therefore 9 \times 16 \cos\theta = -72\sqrt{3}$

$\therefore \cos\theta = \frac{-\sqrt{3}}{2}$ $\therefore \theta = 150°$

$\therefore \vec{A} \times \vec{B} = \pm \|\vec{A}\| \|\vec{B}\| \sin\theta\, \hat{e}$

$= \pm 9 \times 16 \times \sin 150°\, \hat{e} = \pm 72\, \hat{e}$

18 (c)

19 (a)

Solution :

$(2x + 7y)^3 = {}^3c_0 (7y)^0 (2x)^3 + {}^3c_1 (7y)^1 (2x)^2$

$+ {}^3c_2 (7y)^2 (2x) + {}^3c_3 (7y)^3 (2x)^0$

$= 8x^3 + 84yx^2 + 294y^2x + 343y^3$

$\therefore$ the term which has greatest coefficient is T_4

20 (a)

Solution :

$\because$ 2 × coefficient T_2 = coefficient T_1 + coefficient T_3

(dividing by coefficient T_2)

$$\therefore 2 = \frac{\text{coeff. } T_1}{\text{coeff. } T_2} + \frac{\text{coeff. } T_3}{\text{coeff. } T_2}$$

$$\therefore 2 = \frac{1}{n-1+1} \times 2 + \frac{n-2+1}{2} \times \frac{1}{2}$$

$\therefore \frac{2}{n} + \frac{n-1}{4} = 2$ (multiply by 4 n)

$\therefore 8 + n^2 - n = 8n$ $\therefore n^2 - 9n + 8 = 0$

$\therefore (n-8)(n-1) = 0$ $\therefore n = 8$ or $n = 1$ (refused)

21 (d)

Solution :

$$\because |A| = \begin{vmatrix} 1 & 1 & 3 \\ -1 & 0 & 2 \\ 2 & -2 & 1 \end{vmatrix} = 1(0+4) - 1(-1-4) + 3(2-0)$$

$$= 15 \neq 0$$

$\therefore$ RK (A) = 3 $\therefore$ RK (A^t) = RK (A) = 3

22 (a)

Solution :

$$f(x) = \begin{vmatrix} \sin x & 0 & 0 \\ 1 & \sin x & 0 \\ 3 & -2 & \tan 3x \end{vmatrix} = \sin^2 x \tan 3x$$

$$\therefore \lim_{x \to 0} \frac{f(x)}{x^3} = \lim_{x \to 0} \frac{\sin^2 x}{x^2} \times \lim_{x \to 0} \frac{\tan 3x}{x}$$

$= 1 \times 3 = 3$

23 (c)

Solution :

$$\begin{vmatrix} 3a-4\ell & 3b-4m & 3c-4n \\ \ell & m & n \\ x & y & z \end{vmatrix} \text{"doing } R_1 + 4R_2\text{"}$$

$$= \begin{vmatrix} 3a & 3b & 3c \\ \ell & m & n \\ x & y & z \end{vmatrix} = 3 \begin{vmatrix} a & b & c \\ \ell & m & n \\ x & y & z \end{vmatrix}$$

$= 3 \times 24 = 72$

24 (b)

Solution :

$x - ix + y + iy + 2i = 0$

$\therefore (x+y) + i(-x+y+2) = 0$

$\therefore x + y = 0$ and then $x = -y$

, $-x + y + 2 = 0$ $\therefore -x - x = -2$

$\therefore x = 1$, $y = -1$

$\therefore 3x + 3yi = 3 - 4i$

let $a + bi = \sqrt{3-4i}$, a , b $\in \mathbb{R}$

$\therefore a^2 - b^2 + 2abi = 3 - 4i$

$\therefore a^2 - b^2 = 3$ (1) , $ab = -2$

and then $b = \frac{-2}{a}$

$\therefore a^2 - \frac{4}{a^2} = 3$ $\therefore a^4 - 3a^2 - 4 = 0$

$\therefore (a^2 - 4)(a^2 + 1) = 0$

$\therefore a^2 + 1 = 0$ $\therefore a^2 = -1$ (refused)

or $a^2 - 4 = 0$ $\therefore a = 2$, $b = -1$

or $a = -2$, $b = 1$

$\therefore$ results of $\sqrt{3x + 4yi}$ are $\pm (2 - i)$

25 (b)

Solution :

$z = 4\left(\frac{1}{2} + \frac{\sqrt{3}}{2} i\right)^2$

$= 4\left(\cos\frac{\pi}{3} + i \sin\frac{\pi}{3}\right)^2$

$= 4\left(\cos\frac{2\pi}{3} + i \sin\frac{2\pi}{3}\right)$

$\therefore$ sum of principal amplitudes of the roots of

$z = \text{Amp.}(z) = \frac{2\pi}{3}$

Answers of School Book Examinations

Model 1

1

(1) (d) (2) (a) (3) (d) (4) (b) (5) (a) (6) (d)

2

(1) -6048 (2) $\{2\}$

(3) 6 (4) $8\hat{i}-5\hat{j}-6\hat{k}$

(5) $(x-2)^2+(y+3)^2+(z-1)^2=20$

(6) $\vec{r}=(2,-1,4)+t(3,-1,2)$

or $\frac{x-2}{3}=\frac{y+1}{-1}=\frac{z-4}{2}$

3

[a] $T_{r+1}={}^{15}C_r\left(\frac{1}{x^2}\right)^r(2x)^{15-r}$

$={}^{15}C_r(x)^{-2r}(2)^{15-r}(x)^{15-r}$

$={}^{15}C_r(2)^{15-r}x^{15-3r}$

, putting $15-3r=0$ $\therefore r=5$

$\therefore$ The value of the term free of x

is ${}^{15}C_5(2)^{15-5}={}^{15}C_5(2)^{10}=3075072$

, putting $15-3r=5$ $\therefore r=\frac{10}{3}\notin\mathbb{N}$

$\therefore$ There's no term contains x^5

[b] Putting $\frac{x+3}{2}=\frac{2y-1}{5}=\frac{3z+2}{4}=t$

$\therefore x=2t-3$, $y=\frac{5}{2}t+\frac{1}{2}$

, $z=\frac{4}{3}t-\frac{2}{3}$ (parametric equations)

, vector equations :

$\vec{r}=\left(-3,\frac{1}{2},\frac{-2}{3}\right)+t\left(2,\frac{5}{2},\frac{4}{3}\right)$

4

[a] $A=\begin{pmatrix}1 & -1 & 2\\ 2 & -3 & 1\\ 1 & 5 & 21\end{pmatrix}$

$|A|=-1$

The cofactors of B $=\begin{pmatrix}-68 & -41 & 13\\ 31 & 19 & -6\\ 5 & 3 & -1\end{pmatrix}$

$\text{Adj}(A)=\begin{pmatrix}-68 & 31 & 5\\ -41 & 19 & 3\\ 13 & -6 & -1\end{pmatrix}$

$A^{-1}=\begin{pmatrix}68 & -31 & -5\\ 41 & -19 & -3\\ -13 & 6 & 1\end{pmatrix}$

[b] $z=2-2\sqrt{3}\,i$ $\therefore r=\sqrt{(2)^2+(-2\sqrt{3})^2}=4$

$\therefore x>0$, $y<0$ $\therefore \theta$ lies in 4^{th} quadrant

$\therefore \theta=\tan^{-1}\left(\frac{-2\sqrt{3}}{2}\right)=-60°=\frac{-\pi}{3}$

$\therefore z=4\left(\cos\frac{-\pi}{3}+i\sin\frac{-\pi}{3}\right)$

$\therefore$ The square roots of z

$=2\left(\cos\frac{-\frac{\pi}{3}+2\pi n}{2}+i\sin\frac{-\frac{\pi}{3}+2\pi n}{2}\right)$

where $n=0,1$

, at $n=0$

$\therefore$ The 1^{st} square root $=2\left(\cos\frac{-\pi}{6}+i\sin\frac{-\pi}{6}\right)$

, at $n=1$

$\therefore$ The 2^{nd} square root $=2\left(\cos\frac{5\pi}{6}+i\sin\frac{5\pi}{6}\right)$

5

[a] The matrix equation :

$\begin{pmatrix}1 & 3 & 2\\ 2 & -1 & 1\\ 3 & 1 & -1\end{pmatrix}\begin{pmatrix}x\\ y\\ z\end{pmatrix}=\begin{pmatrix}13\\ 3\\ 2\end{pmatrix}$

$A=\begin{pmatrix}1 & 3 & 2\\ 2 & -1 & 1\\ 3 & 1 & -1\end{pmatrix}$ $\therefore |A|=25$

$\text{Adj}(A)=\begin{pmatrix}0 & 5 & 5\\ 5 & -7 & 3\\ 5 & 8 & -7\end{pmatrix}$

$A^{-1}=\frac{1}{25}\begin{pmatrix}0 & 5 & 5\\ 5 & -7 & 3\\ 5 & 8 & -7\end{pmatrix}$

$\therefore \begin{pmatrix}x\\ y\\ z\end{pmatrix}=\frac{1}{25}\begin{pmatrix}0 & 5 & 5\\ 5 & -7 & 3\\ 5 & 8 & -7\end{pmatrix}\begin{pmatrix}13\\ 3\\ 2\end{pmatrix}=\begin{pmatrix}1\\ 2\\ 3\end{pmatrix}$

$\therefore x=1$, $y=2$, $z=3$

[b] To get the point of intersection between 3 planes, the cramers method can be used to find the solution

$$\Delta = \begin{vmatrix} 2 & 1 & -1 \\ 1 & 1 & 1 \\ 3 & -1 & -1 \end{vmatrix}$$

$= 2 \times (0) - 1 \times -4 - 1 \times -4 = 8$

$$, \Delta_x = \begin{vmatrix} -1 & 1 & -1 \\ 2 & 1 & 1 \\ 6 & -1 & -1 \end{vmatrix}$$

$= -1 \times 0 - 1 \times -8 - 1 \times -8 = 16$

$$, \Delta_y = \begin{vmatrix} 2 & -1 & -1 \\ 1 & 2 & 1 \\ 3 & 6 & -1 \end{vmatrix}$$

$= 2 \times -8 + 1 \times -4 - 1 \times 0 = -20$

$$, \Delta_z = \begin{vmatrix} 2 & 1 & -1 \\ 1 & 1 & 2 \\ 3 & -1 & 6 \end{vmatrix}$$

$= 2 \times 8 - 1 \times 0 - 1 \times -4 = 20$

$x = \frac{16}{8} = 2$, $y = \frac{-20}{8} = -\frac{5}{2}$, $z = \frac{20}{8} = \frac{5}{2}$

∴ The point of intersection between the three planes is : $\left(2 , \frac{-5}{2} , \frac{5}{2}\right)$

Model 2

1

(1) (c)	(2) (d)	(3) (a)
(4) (a)	(5) (b)	(6) (a)

2

(1) ω	(2) 0	(3) $\frac{8}{3}$
(4) 0 or $\frac{1}{2}$	(5) -2	(6) -33

3

[a] T_{r+1} is the general term in the expansion of $(1 + x)^{11} = {}^{11}c_r \, x^r$

$\therefore (1 - x + x^2) \times {}^{11}c_r \, x^r$

$= {}^{11}c_r \, x^r - {}^{11}c_r \, x^{r+1} + {}^{11}c_r \, x^{r+2}$

∴ The 1st part contains x^5

, putting $r = 5$, its value ${}^{11}c_5$

∴ 2nd part contains x^5

, putting $r + 1 = 5$ ∴ $r = 4$, its value ${}^{11}C_4$

, 3rd part contains x^5 , putting $r + 2 = 5$

∴ $r = 3$, its value ${}^{11}C_3$

∴ The coefficient of $x^5 = {}^{11}C_5 - {}^{11}C_4 + {}^{11}C_3 = 297$

[b] ∵ $(2 , -1 , 3)$ is the direction vector of the straight line

, $(3 , 2 , 1)$ is the normal direction vector of the plane.

, ∵ $(2 , -1 , 3) \cdot (3 , 2 , 1) = 7 \neq 0$

∴ The straight line is not parallel to the plane

∴ The straight line intesects the plane

$$\therefore \cos\theta = \frac{|(2 , -1 , 3) \cdot (3 , 2 , 1)|}{\sqrt{4+1+9}\,\sqrt{9+4+1}} = \frac{1}{2}$$

∴ $m(\angle\theta) = 60°$ is the angle between the direction vector of the straight line and the normal vector of the plane.

∴ The measure of the minor angle between the straight line and the plane $= 90° - 60° = 30°$

4

[a] The matrix $A = \begin{pmatrix} 2 & -1 & -3 \\ 1 & 2 & 1 \\ 3 & -5 & 2 \end{pmatrix}$

$|A| \; 50 \neq 0$ ∴ RK (A) = 3

$$, A^* = \left(\begin{array}{ccc|c} 2 & -1 & -3 & 2 \\ 1 & 2 & 1 & 1 \\ 3 & -5 & 2 & 13 \end{array}\right)$$

∴ RK (A) = RK (A^*) = 3 = the number of variable

∴ The equations has a unique solution

$$A^{-1} = \frac{1}{50}\begin{pmatrix} 9 & 17 & 5 \\ 1 & 13 & -5 \\ -11 & 7 & 5 \end{pmatrix}$$

$$\therefore \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{50}\begin{pmatrix} 9 & 17 & 5 \\ 1 & 13 & -5 \\ -11 & 7 & 5 \end{pmatrix}\begin{pmatrix} 2 \\ 1 \\ 13 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$$

∴ $x = 2$, $y = -1$, $z = 1$

[b] $z = \frac{2 + 6i}{3 - i} \times \frac{3 + i}{3 + i} = 2i = 2e^{\frac{\pi}{2}i}$

$\therefore z = 2\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$

$z^{-1} = \frac{1}{2}\left(\cos\frac{-\pi}{2} + i\sin\frac{-\pi}{2}\right)$

$\bar{z} = 2\left(\cos\frac{-\pi}{2} + i\sin\frac{-\pi}{2}\right)$

$$\sqrt{z} = \sqrt{2}\left(\cos\frac{\frac{\pi}{2} + 2\pi n}{2} + i\sin\frac{\frac{\pi}{2} + 2\pi n}{2}\right)$$

where $n = 0 , 1$

, at $n = 0$

∴ The 1st square root $= \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

, at $n = 1$

∴ The 2nd square root $= \sqrt{2}\left(\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}\right)$

$= \sqrt{2}\left(\cos\frac{-3\pi}{4} + i\sin\frac{-3\pi}{4}\right)$

5

[a] ∵ $\sqrt{i} = (\cos 90° + i\sin 90°)^{\frac{1}{2}}$

$= \cos\frac{90 + 360n}{2} + i\sin\frac{90 + 360n}{2}$

where $n = 0 , 1$

, at $n = 0$ ∴ $\sqrt{i} = \cos 45° + i\sin 45° = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} i$

, at $n = 1$ ∴ $\sqrt{i} = \cos 225° + i\sin 225° = \frac{-1}{\sqrt{2}} - \frac{1}{\sqrt{2}} i$

$\therefore \sqrt{i} = \pm\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} i\right)$

, ∵ $\sqrt{-i} = (\cos(-90°) + i\sin(-90°))$

$= \cos\frac{-90 + 360n}{2} + i\sin\frac{-90 + 360n}{2}$

where $n = 0 , 1$

, at $n = 0$

$\therefore \sqrt{-i} = \cos(-45°) + i\sin(-45°) = \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} i$

, at $n = 1$

$\therefore \sqrt{-i} = \cos(135°) + i\sin(135°) = \frac{-1}{\sqrt{2}} + \frac{1}{\sqrt{2}} i$

$\therefore \sqrt{-i} = \pm\left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} i\right)$

$\therefore \sqrt{i} - \sqrt{-i} = \pm\sqrt{2}\, i$ or $\pm\sqrt{2}$

∴ $\sqrt{2}\, i$ is one of the values of $\sqrt{i} - \sqrt{-i}$

[b] The centres of the two spheres are :

$(2 , -4 , 2) , (-4 , 4 , 2)$

The distance between the two centres

$= \sqrt{(2 + 4)^2 + (-4 - 4)^2 + (2 - 2)^2}$

∴ $MN = 10$ $r_1 = 1$, $r_2 = 2$

$r_1 + r_2 < MN$

∴ The two spheres do not intersect.

Model 3

1

(1) (c)	(2) (b)	(3) (a)
(4) (d)	(5) (b)	(6) (a)

2

(1) $-30°$ or $\frac{-\pi}{6}$	(2) 1	(3) 3
(4) 5	(5) $\frac{-21}{2}$	(6) 12

3

[a] The expansion of $(m + x)^n$

$= m^n + n\, m^{n-1} x + \frac{n(n-1)}{2 \times 1} m^{n-2} x^2 + \ldots\ldots$

By comparing the coefficients of free of x :

$\therefore m^n = 3a$ (1)

, comparing the coefficients of x : $n\, m^{n-1} = 6a$ (2)

Divide (2) ÷ (1) : ∴ $n\, m^{-1} = 2$

$\therefore \frac{n}{m} = 2$ $\therefore m = \frac{n}{2}$

By comparing the coefficients of x^2 :

$\frac{n(n-1)}{2 \times 1} m^{n-2} = 5a$ (3)

Divide (3) ÷ (1) : ∴ $\frac{n(n-1)}{2 \times 1} m^{-2} = \frac{5}{3}$

$\therefore \frac{n(n-1)}{2 \times 1} \times \left(\frac{n}{2}\right)^{-2} = \frac{5}{3}$ $\therefore \frac{n(n-1) \times 4}{2n^2} = \frac{5}{3}$

$\therefore 12(n^2 - n) = 10n^2$ $\therefore 12n^2 - 12n = 10n^2$

$\therefore 2n^2 - 12n = 0$ $2n(n - 6) = 0$

$\therefore n = 6$, $m = 3$, $a = \frac{3^6}{3} = 3^5 = 243$

[b] ∵ $A = \begin{pmatrix} 2 & -1 & 3 \\ 4 & 5 & -1 \\ 2 & 3 & -1 \end{pmatrix}$

$$\therefore |A| = \begin{vmatrix} 2 & -1 & 3 \\ 4 & 5 & -1 \\ 2 & 3 & -1 \end{vmatrix} = 0$$

∴ RK (A) < 3 (less than the number of variables)

∴ The equations have an infinite number of solutions one of them the zero solution.

i.e. The equations have a solution other than the zero solution.

put $z = \ell$ $\therefore 2x - y = -3\ell , 4x + 5y = \ell$

By solving the two equations

$\therefore x = \ell , y = \ell$ ∴ The general form is $(-\ell , \ell , \ell)$

4

[a] The arg $(z_1 z_2^3)$

$= (\arg \text{ of } z_1) + 3 \times (\arg \text{ of } z_2) = 81°$ (1)

, the arg of $\left(\frac{z_1}{z_2}\right) = (\arg \text{ of } z_1)$

$-(\arg \text{ of } z_2) = 33°$ (2)

From (1) , (2) :

$\arg \text{ of } z_1 = 45°$, $\arg \text{ of } z_2 = 12°$

$\therefore z_1 = \cos 45° + i \sin 45°$, $z_2 = \cos 12° + i \sin 12°$

$\therefore z_1^{15} z_2^{15} = (z_1 z_2)^{15}$

$= (\cos 57° + i \sin 57°)^{15}$

$= \cos 135° + i \sin 135° = \frac{-1}{\sqrt{2}} + \frac{1}{\sqrt{2}} i$

[b] $\because$ The point $A(-2, 3, 1) \in$ the straight line

$\therefore$ The length of perpendicular $= 0$

5

[a] R.H.S. $= \begin{vmatrix} a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \\ 1 & 1 & 1 \end{vmatrix}$

by taking a , b , c as a common factor from c_1 , c_2 , c_3 respectively

$= abc \begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ \frac{1}{a} & \frac{1}{b} & \frac{1}{c} \end{vmatrix}$

multiply $(abc) \times R_3 = \begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ bc & ac & ab \end{vmatrix} =$ L.H.S.

[b] $\overrightarrow{BD} = \overrightarrow{D} - \overrightarrow{B} = (0, 0, 0) - (4, 8, 6)$

$= (-4, -8, -6)$

$\overrightarrow{CA} = \overrightarrow{A} - \overrightarrow{C} = (0, 8, 0) - (4, 0, 6)$

$= (-4, 8, -6)$

$\overrightarrow{BD} \cdot \overrightarrow{CA} = (-4, -8, -6) \cdot (-4, 8, -6)$

$= 16 - 64 + 36 = -12$

Model 4

1

(1) (b) (2) (a) (3) (d)

(4) (c) (5) (a) (6) (c)

2

(1) -40 (2) 2 (3) $(-4, 6, -1)$

(4) 100 (5) $\left(\frac{2}{5}, \frac{3}{5}, \frac{2}{5}\sqrt{3}\right)$

(6) $\sqrt{10}$ length unit.

3

[a] Let T_{r+1} is the greatest term $\frac{T_{r+1}}{T_r} \geq 1$

$\frac{6-r+1}{r} \left|\frac{2(1)}{3}\right| \geq 1$, $\frac{2(7-r)}{3r} \geq 1$

$14 - 2r \geq 3r$ $\therefore 5r \leq 14$

, $r \leq 2\frac{4}{5}$ $\therefore r = 2$ or 1 or 0

$\therefore T_3 > T_2 > T_1$, $\frac{T_{r+1}}{T_{r+2}} \geq 1$

$\therefore \frac{T_{r+2}}{T_{r+1}} \leq 1$ $\therefore \frac{6-(r+1)+1}{r+1} \times \left|\frac{2(1)}{3}\right| \leq 1$

$\therefore \frac{2(6-r)}{3(r+1)} \leq 1$ $\therefore 12 - 2r \leq 3r + 3$

$\therefore 5r \geq 9$ $\therefore r \geq 1\frac{4}{5}$

$\therefore r = 2$ or 3 or ..... or 6

$\therefore T_3 > T_4 > T_5 > T_6 > T_7$

$\therefore T_3$ is the greatest term

[b] $\because \overrightarrow{A} \cdot \overrightarrow{B} \times \overrightarrow{C} = \begin{vmatrix} 1 & -1 & 2 \\ 3 & -2 & 0 \\ 0 & 2 & 4 \end{vmatrix} = 16$

$\therefore$ Volume of the parallelepiped $= |16|$

$= 16$ cubic unit.

4

[a] $z^4 = -4 = 4(\cos 180° + i \sin 180°)$

$\therefore z = 4^{\frac{1}{4}} \left(\cos \frac{180° + 360° n}{4} + i \sin \frac{180° + 360° n}{4}\right)$

where $n = 0, 1, 2, 3$

, at $n = 0$ $\therefore z = \sqrt{2}(\cos 45° + i \sin 45°)$

, at $n = 1$ $\therefore z = \sqrt{2}(\cos 135° + i \sin 135°)$

, at $n = 2$ $\therefore z = \sqrt{2}(\cos -135° + i \sin -135°)$

, at $n = 3$ $\therefore z = \sqrt{2}(\cos -45° + i \sin -45°)$

[b] (1) $\|2\overrightarrow{A} - \overrightarrow{B} + 3\overrightarrow{C}\|^2$

$= (2\overrightarrow{A} - \overrightarrow{B} + 3\overrightarrow{C}) \cdot (2\overrightarrow{A} - \overrightarrow{B} + 3\overrightarrow{C})$

$= 4\|\overrightarrow{A}\|^2 - 2\overrightarrow{A} \cdot \overrightarrow{B} + 6\overrightarrow{A} \cdot \overrightarrow{C} - 2\overrightarrow{A} \cdot \overrightarrow{B}$

$+ \|\overrightarrow{B}\|^2 - 3\overrightarrow{B} \cdot \overrightarrow{C} + 6\overrightarrow{A} \cdot \overrightarrow{C} - 3\overrightarrow{B} \cdot \overrightarrow{C}$

$+ 9\|\overrightarrow{C}\|^2 = 4 - 0 + 0 - 0 + 1 - 0 + 0 - 0 + 9 = 14$

$\therefore \|2\overrightarrow{A} - \overrightarrow{B} + 3\overrightarrow{C}\| = \sqrt{14}$

(2) $\overrightarrow{C} = \overrightarrow{A} \times \overrightarrow{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{16}{25} & \frac{-3}{5} & \frac{12}{25} \\ \frac{3}{5} & 0 & \frac{-4}{5} \end{vmatrix}$

$= \frac{12}{25}\hat{i} + \frac{4}{5}\hat{j} + \frac{9}{25}\hat{k}$

5

[a] $x + y = 2$, $2x + 3y = 5$

$\therefore$ Matrix equation : $\begin{pmatrix} 1 & 1 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$

$\therefore A = \begin{pmatrix} 1 & 1 \\ 2 & 3 \end{pmatrix}$, $A^* = \left(\begin{array}{cc|c} 1 & 1 & 2 \\ 2 & 3 & 5 \end{array}\right)$

$\therefore RK(A) = 2$, $RK(A^*) = 2$

$\therefore RK(A) = RK(A^*) =$ the number of variables

$\therefore$ There is a unique solution , to get the solution :

$A^{-1} = \frac{1}{1}\begin{pmatrix} 3 & -1 \\ -2 & 1 \end{pmatrix} = \begin{pmatrix} 3 & -1 \\ -2 & 1 \end{pmatrix}$

$\therefore \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 & -1 \\ -2 & 1 \end{pmatrix}\begin{pmatrix} 2 \\ 5 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$\therefore x = 1$, $y = 1$

[b] $z = \sin \frac{\pi}{9} + i \cos \frac{\pi}{9}$

$\therefore z = \cos\left(\frac{\pi}{2} - \frac{\pi}{9}\right) + i \sin\left(\frac{\pi}{2} - \frac{\pi}{9}\right)$

$= \cos\left(\frac{7\pi}{18}\right) + i \sin\left(\frac{7\pi}{18}\right)$

$\therefore \bar{z} = \cos\left(\frac{-7\pi}{18}\right) + i \sin\left(\frac{-7\pi}{18}\right)$

$\therefore (\bar{z})^9 = \cos\left(\frac{-7\pi}{2}\right) + i \sin\left(\frac{-7\pi}{2}\right)$

$= \cos\left(\frac{\pi}{2}\right) + i \sin\left(\frac{\pi}{2}\right)$

$\therefore$ The cubic roots of $(\bar{z})^9$

$= \cos\left(\frac{\frac{\pi}{2} + 2\pi n}{3}\right) + i \sin\left(\frac{\frac{\pi}{2} + 2\pi n}{3}\right)$

where $n = 0, 1, 2$

, at $n = 0$

$\therefore$ The 1st cubic root $= \cos\left(\frac{\pi}{6}\right) + i \sin\left(\frac{\pi}{6}\right)$

, at $n = 1$

$\therefore$ The 2nd cubic root $= \cos\left(\frac{5\pi}{6}\right) + i \sin\left(\frac{5\pi}{6}\right)$

, at $n = 2$

The 3rd cubic root $= \cos\left(\frac{3\pi}{2}\right) + i \sin\left(\frac{3\pi}{2}\right)$

$= \cos\left(\frac{-\pi}{2}\right) + i \sin\left(\frac{-\pi}{2}\right)$

Model 5

1

(1) (b) (2) (d) (3) (a)

(4) (b) (5) (d) (6) (c)

2

(1) 133 (2) 20 (3) $\frac{1}{9}$

(4) 3 (5) $(x-3)^2 + (y-4)^2 + (z+5)^2 = 9$

(6) $\overrightarrow{r} = (2, -1, 4) + t(4, 7, 1)$

3

[a] The coefficient of $T_{2r+4} = {}^{18}C_{2r+3}$

, the coefficient of $T_{r-2} = {}^{18}C_{r-3}$

$\because {}^{18}C_{2r+3} = {}^{18}C_{r-3}$

$\therefore 2r + 3 = r - 3$ then $r = -6$ (refused)

or $2r + 3 + r - 3 = 18$ then $3r = 18$ $\therefore r = 6$

[b] $\because \frac{|\sqrt{2}(0) + (-1) - (2) + k|}{\sqrt{2+1+1}} = 2$ $\therefore |k - 3| = 4$

$k - 3 = \pm 4$ $\therefore k = 7$ or $k = -1$

4

[a] The matrix equation :

$\begin{pmatrix} 2 & 1 & -2 \\ 1 & 2 & 2 \\ 5 & 4 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 10 \\ 1 \\ 6 \end{pmatrix}$

$$\therefore A=\begin{pmatrix}2&1&-2\\1&2&2\\5&4&3\end{pmatrix} \qquad \therefore |A|=15$$

$$\therefore \text{Adj}(A)=\begin{pmatrix}-2&-11&6\\7&16&-6\\-6&-3&3\end{pmatrix}$$

$$\therefore A^{-1}=\frac{1}{15}\begin{pmatrix}-2&-11&6\\7&16&-6\\-6&-3&3\end{pmatrix}$$

$$\therefore \begin{pmatrix}x\\y\\z\end{pmatrix}=\frac{1}{15}\begin{pmatrix}-2&-11&6\\7&16&-6\\-6&-3&3\end{pmatrix}\begin{pmatrix}10\\1\\6\end{pmatrix}=\begin{pmatrix}\frac{1}{3}\\\frac{10}{3}\\-3\end{pmatrix}$$

$\therefore x=\frac{1}{3}$, $y=\frac{10}{3}$, $z=-3$

[b] $z_1=\frac{6+4i}{1+i}\times\frac{1-i}{1-i}=5-i$

$, z_2=\frac{26}{5-i}\times\frac{5+i}{5+i}=5+i$

$\therefore z_1-z_2=(5-i)-(5+i)=-2i$

$\therefore 4(z_1-z_2)=-8i$

$=8\left(\cos\frac{-\pi}{2}+i\sin\frac{-\pi}{2}\right)$

$\therefore$ The cubic roots of $4(z_1-z_2)$

$$=2\left(\cos\frac{\frac{-\pi}{2}+2\pi n}{3}+i\sin\frac{\frac{-\pi}{2}+2\pi n}{3}\right)$$

where $n=0 , 1 , 2$

, at $n=0$

$\therefore$ The 1st cubic root $=2\left(\cos\frac{-\pi}{6}+i\sin\frac{-\pi}{6}\right)=2e^{\frac{-\pi}{6}i}$

, at $n=1$

$\therefore$ The 2nd cubic root $=2\left(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}\right)=2e^{\frac{\pi}{2}i}$

, at $n=2$

$\therefore$ The 3rd cubic root $=2\left(\cos\frac{-5\pi}{6}+i\sin\frac{-5\pi}{6}\right)$
$=2e^{\frac{-5\pi}{6}i}$

5

[a] $$\text{L.H.S.}=\begin{vmatrix}a^2+1&ab&ac\\ab&b^2+1&bc\\ac&bc&c^2+1\end{vmatrix}$$

$$=\begin{vmatrix}a^2&ab&ac\\ab&b^2+1&bc\\ac&bc&c^2+1\end{vmatrix}$$

$$+\begin{vmatrix}1&ab&ac\\0&b^2+1&bc\\0&bc&c^2+1\end{vmatrix}$$

by taking (a) a common factor from each of R_1 , C_1 in the 1st determinant.

$$=a^2\begin{vmatrix}1&b&c\\b&b^2+1&bc\\c&bc&c^2+1\end{vmatrix}$$

$$+\begin{vmatrix}1&ab&ac\\0&b^2&bc\\0&bc&c^2+1\end{vmatrix}+\begin{vmatrix}1&0&ac\\0&1&bc\\0&0&c^2+1\end{vmatrix}$$

(by doing R_2-bR_1) , (R_3-cR_1)

in the 1st determinant , (b) a common factor from , R_2 , C_2 in the 2nd determinant

$$=a^2\begin{vmatrix}1&b&c\\0&1&0\\0&0&1\end{vmatrix}+b^2\begin{vmatrix}1&a&ac\\0&1&c\\0&c&c^2+1\end{vmatrix}$$

$+(c^2+1)$ (by doing R_3-cR_2)

in the 2nd determinant

$$=a^2+b^2\begin{vmatrix}1&a&ac\\0&1&c\\0&0&1\end{vmatrix}+(c^2+1)$$

$=a^2+b^2+c^2+1=$ R.H.S.

Another solution :

$$\text{L.H.S.}=\begin{vmatrix}a^2+1&ab&ac\\ab&b^2+1&bc\\ac&bc&c^2+1\end{vmatrix}$$

by taking "a" common factor from R_1 , "b" from R_2 , "c" from R_3 then $a\times c_1$, $b\times c_2$, $c\times c_3$

$$=\begin{vmatrix}a^2+1&b^2&c^2\\a^2&b^2+1&c^2\\a^2&b^2&c^2+1\end{vmatrix} , c_1+(c_2+c_3)$$

$$=\begin{vmatrix}a^2+b^2+c^2+1&b^2&c^2\\a^2+b^2+c^2+1&b^2+1&c^2\\a^2+b^2+c^2+1&b^2&c^2+1\end{vmatrix}$$

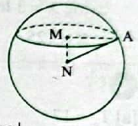
$$=(a^2+b^2+c^2+1)\begin{vmatrix}1&b^2&c^2\\1&b^2+1&c^2\\1&b^2&c^2+1\end{vmatrix}$$

(R_2-R_1) , (R_3-R_1)

$$=(a^2+b^2+c^2+1)\begin{vmatrix}1&b^2&c^2\\0&1&0\\0&0&1\end{vmatrix}$$

$=(a^2+b^2+c^2+1)=$ R.H.S.

[b] The centre of the sphere



$N(-3 , -2 , 1)$

, its radius $AN=\sqrt{15}$

$\because MN=\frac{|2(-3)-(-2)-2(1)+12|}{\sqrt{4+1+4}}$
$=2$ length unit.

$\therefore$ r of the circle $=\sqrt{15-4}=\sqrt{11}$ length unit.

$\therefore$ The area of the cross section
$=11\pi$ square unit.

Model 6

1

(1) (c)	(2) (a)	(3) (c)
(4) (c)	(5) (d)	(6) (b)

2

(1) 243	(2) 3	(3) (3 , 0 , 2)
(4) 1 or $\frac{-13}{5}$	(5) -5	(6) -41

3

[a] $\because$ The coefficients of T_4 , T_5 , T_6 form an arithmetic sequence

$\therefore 2T_5=T_4+T_6$, divide by T_5

$2=\frac{T_4}{T_5}+\frac{T_6}{T_5}$

$2=\frac{4}{n-4+1}\times 2+\frac{n-5+1}{5}\times\frac{1}{2}$

$2=\frac{8}{n-3}+\frac{n-4}{10}$ multiply $\times 10(n-3)$

$20(n-3)=80+(n-3)(n-4)$

$n^2-27n+152=0$ $\qquad$ $n=19$ or $n=8$

[b] r of the sphere = the length of the perpendicular drawn from its centre to the plane.

$\therefore r=\frac{|1+2+1-1|}{\sqrt{1+1+1}}=\sqrt{3}$ length unit.

$\therefore$ The equation of the sphere is :

$(x-1)^2+(y-2)^2+(z-1)^2=3$

4

[a] The matrix equation :

$$\begin{pmatrix}4&3&-5\\3&2&4\\5&-2&-7\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}6\\12\\1\end{pmatrix}$$

$$A=\begin{pmatrix}4&3&-5\\3&2&4\\5&-2&-7\end{pmatrix} \qquad \therefore |A|=179$$

$$\text{Adj}(A)=\begin{pmatrix}-6&31&22\\41&-3&-31\\-16&23&-1\end{pmatrix}$$

$$A^{-1}=\frac{1}{179}\begin{pmatrix}-6&31&22\\41&-3&-31\\-16&23&-1\end{pmatrix}$$

$$\therefore\begin{pmatrix}x\\y\\z\end{pmatrix}=\frac{1}{179}\begin{pmatrix}-6&31&22\\41&-3&-31\\-16&23&-1\end{pmatrix}\begin{pmatrix}6\\12\\1\end{pmatrix}=\begin{pmatrix}2\\1\\1\end{pmatrix}$$

$\therefore x=2$, $y=1$, $z=1$ $\qquad \therefore$ S.S. $=\{(2 , 1 , 1)\}$

[b] $z_1=\left(\frac{\sqrt{3}+i}{2}\right)^4=\left[\left(\frac{\sqrt{3}+i}{2}\right)^2\right]^2$

$=\left(\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)^2=\frac{-1}{2}+\frac{\sqrt{3}}{2}i$

$, z_2=\sin\frac{\pi}{3}+i\cos\frac{\pi}{3}=\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}=\frac{\sqrt{3}}{2}+\frac{1}{2}i$

$$\therefore z=\frac{-\frac{1}{2}+\frac{\sqrt{3}}{2}i}{\frac{\sqrt{3}}{2}+\frac{1}{2}i}\times\frac{\frac{\sqrt{3}}{2}-\frac{1}{2}i}{\frac{\sqrt{3}}{2}-\frac{1}{2}i}=i$$

$=\cos\left(\frac{\pi}{2}\right)+i\sin\left(\frac{\pi}{2}\right)$

The square roots of the numbers z

$$=\cos\left(\frac{\frac{\pi}{2}+2\pi n}{2}\right)+i\sin\left(\frac{\frac{\pi}{2}+2\pi n}{2}\right)$$

where $n=0 , 1$

, at $n=0$

$\therefore$ The 1st square root $=\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}$

, at $n=1$

$\therefore$ The 2nd square root $=\cos\frac{5\pi}{4}+i\sin\frac{5\pi}{4}$

$=\cos\frac{-3\pi}{4}+i\sin\frac{-3\pi}{4}$

5

[a] L.H.S. $= \begin{vmatrix} x & a & b \\ a & x & b \\ b & a & x \end{vmatrix}$

by doing $(C_1 + C_2 + C_3)$

then taking $(x + a + b)$ common factor from C_1

$= (x + a + b) \begin{vmatrix} 1 & a & b \\ 1 & x & b \\ 1 & a & x \end{vmatrix}$

(by doing $R_2 - R_1$, $R_3 - R_1$)

$= (x + a + b) \begin{vmatrix} 1 & a & b \\ 0 & x-a & 0 \\ 0 & 0 & x-b \end{vmatrix}$

$= (x + a + b)(x - a)(x - b) =$ R.H.S.

[b] Direction vector $= (5 , 2 , -3)$

The vector equation :

$\vec{r} = (2 , 1 , -3) + t(5 , 2 , -3)$

, the Parametric equation :

$x = 2 + 5t$, $y = 1 + 2t$, $z = -3 - 3t$

, the Cartesian equation : $\frac{x-2}{5} = \frac{y-1}{2} = \frac{z+3}{-3}$

Model 7

1

(1) (b) (2) (c) (3) (a)
(4) (c) (5) (d) (6) (a)

2

(1) 1 (2) 3 (3) 45°
(4) 8 (5) −1 (6) $\sqrt{157}$

3

[a] $z_1 = \left(\cos \frac{7\pi}{18} + i \sin \frac{7\pi}{18}\right)^5$

$= \cos \frac{35\pi}{18} + i \sin \frac{35\pi}{18} = \cos \frac{-\pi}{18} + i \sin \frac{-\pi}{18}$

, $z_2 = \left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}\right)^4 = (\cos 0 + i \sin 0)$

$\therefore z = \frac{z_1}{z_2} = \cos \frac{-\pi}{18} + i \sin \frac{-\pi}{18} = e^{\frac{-\pi}{18} i}$

$\sqrt{z} = e^{\frac{\left(\frac{-\pi}{18} + 2\pi n\right) i}{2}}$ where $n = 0 , 1$

, at $n = 0$ $\therefore \sqrt{z} = e^{\frac{-\pi}{36} i}$, at $n = 1$ $\therefore \sqrt{z} = e^{\frac{35\pi}{36} i}$

[b] $\vec{A} \cdot \vec{B} = 2\cos^2\theta + \log_3 x \cdot \log_5 27 + 2\sin^2\theta = 11$

$\therefore \log_3 x \log_5 27 = 9$ $\therefore \frac{\log x}{\log 3} \cdot \frac{\log 27}{\log 5} = 9$

$\therefore \frac{\log x}{\log 3} \cdot \frac{3 \log 3}{\log 5} = 9$ $\therefore \frac{\log x}{\log 5} = 3$

$\therefore \log x = 3 \log 5$ $\therefore \log x = \log 5^3$

$\therefore x = 125$

4

[a] $T_3 = 17$ ${}^nC_2 \times x^2 = 17$

$n(n-1)x^2 = 34$ (1)

$3 T_2 \times T_4 = 544$ $3 \times {}^nC_1 x \times {}^nC_3 x^3 = 544$

$3n \times \frac{n(n-1)(n-2)}{3 \times 2 \times 1} x^4 = 544$

$n^2(n-1)(n-2)x^4 = 1088$ (2)

divide (2) ÷ the square of (1)

$\frac{\cancel{n^2}(n-1)(n-2)x^4}{\cancel{n^2}(n-1)^2 x^4} = \frac{1088}{(34)^2}$ $\frac{n-2}{n-1} = \frac{16}{17}$

$\therefore n = 18$

by substitute in (1) : $18 \times 17 x^2 = 34$

$\therefore x^2 = \frac{1}{9}$ $\therefore x = \pm\frac{1}{3}$

[b] R.H.S. $= \begin{vmatrix} a+b+2 & a & b \\ 1 & 2a+b+1 & b \\ 1 & a & a+2b+1 \end{vmatrix}$

by doing $(C_1 + C_2 + C_3)$ then taking

$2(a + b + 1)$ common factor from C_1

$= 2(a+b+1) \begin{vmatrix} 1 & a & b \\ 1 & 2a+b+1 & b \\ 1 & a & a+2b+1 \end{vmatrix}$

(by doing $R_2 - R_1$, $R_3 - R_1$)

$= 2(a+b+1) \begin{vmatrix} 1 & a & b \\ 0 & a+b+1 & 0 \\ 0 & 0 & a+b+1 \end{vmatrix}$

$= 2(a + b + 1)^3 =$ R.H.S.

5

[a] $\because A^t = A^{-1}$, multiplying × A from the right side

$\therefore A^t A = A^{-1} A = I$

$\therefore \begin{pmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{pmatrix} \begin{pmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{pmatrix} = I$

$\therefore \begin{pmatrix} 2x^2 & 0 & 0 \\ 0 & 6y^2 & 0 \\ 0 & 0 & 3z^2 \end{pmatrix} = I$ $\therefore 2x^2 = 1$

$\therefore x = \pm\frac{1}{\sqrt{2}}$, $6y^2 = 1$ $\therefore y = \pm\frac{1}{\sqrt{6}}$

, $3z^2 = 1$ $\therefore z = \pm\frac{1}{\sqrt{3}}$

[b] Let $x = y = z = t$

by substitute in the equation of the plane

$\therefore t + 2t + 3t = 12$ $\therefore 6t = 12$ $\therefore t = 2$

$\therefore$ The point is (2 , 2 , 2)

Model 8

1

(1) $\frac{1}{10}$ or 1 (2) 5 (3) 90°
(4) − 16 (5) $(x-5)^2 + y^2 + (z+1)^2 = 14$
(6) 11 or − 1

2

(1) (a) (2) (b) (3) (d)
(4) (a) (5) (b) (6) (b)

3

[a] The matrix equation is :

$\begin{pmatrix} 2 & -1 & 1 \\ 1 & 0 & -1 \\ 1 & 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix}$

$|A| = \begin{vmatrix} 2 & -1 & 1 \\ 1 & 0 & -1 \\ 1 & 1 & 0 \end{vmatrix} = 4$

$\therefore A^{-1} = \frac{1}{4} \begin{pmatrix} 1 & 1 & 1 \\ -1 & -1 & 3 \\ 1 & -3 & 1 \end{pmatrix}$

$\therefore \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{4} \begin{pmatrix} 1 & 1 & 1 \\ -1 & -1 & 3 \\ 1 & -3 & 1 \end{pmatrix} \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix}$

$\therefore x = 1$, $y = 2$, $z = -1$

[b] To get the point of intersection between 3 planes

, the cramers method can be used to find the solution

$\Delta = \begin{vmatrix} 2 & 1 & -1 \\ 1 & 1 & 1 \\ 3 & -1 & -1 \end{vmatrix} = 2 \times (0) - 1 \times -4 - 1 \times -4 = 8$

, $\Delta_x = \begin{vmatrix} -1 & 1 & -1 \\ 2 & 1 & 1 \\ 6 & -1 & -1 \end{vmatrix} = -1 \times 0 - 1 \times -8 - 1 \times -8 = 16$

, $\Delta_y = \begin{vmatrix} 2 & -1 & -1 \\ 1 & 2 & 1 \\ 3 & 6 & -1 \end{vmatrix} = 2 \times -8 + 1 \times -4 - 1 \times 0 = -20$

, $\Delta_z = \begin{vmatrix} 2 & 1 & -1 \\ 1 & 1 & 2 \\ 3 & -1 & 6 \end{vmatrix} = 2 \times 8 - 1 \times 0 - 1 \times -4 = 20$

$x = \frac{16}{8} = 2$, $y = \frac{-20}{8} = \frac{-5}{2}$, $z = \frac{20}{8} = \frac{5}{2}$

$\therefore$ The point of intersection between the three planes is : $\left(2 , \frac{-5}{2} , \frac{5}{2}\right)$

4

[a] $z_1 = 1 - \sqrt{3} i$

$\therefore r = \sqrt{(1)^2 + (-\sqrt{3})^2} = 2$ $\because x > 0 , y < 0$

$\therefore \theta$ lies in the fourth quadrant

$\therefore \theta = \tan^{-1}\left(\frac{-\sqrt{3}}{1}\right) = -60° = \frac{-\pi}{3}$

$\therefore z_1 = 2\left(\cos \frac{-\pi}{3} + i \sin \frac{-\pi}{3}\right)$

$z_3 = \left(\cos \frac{\theta}{2} - i \sin \frac{\theta}{2}\right)^2 = \left(\cos \frac{-\theta}{2} + i \sin \frac{-\theta}{2}\right)^2$

$= \cos(-\theta) + i \sin(-\theta)$

$\therefore z = \frac{\left(2\left(\cos -\frac{\pi}{3} + i \sin -\frac{\pi}{3}\right)\right)(\cos\theta + i \sin\theta)}{\cos(-\theta) + i \sin(-\theta)}$

$= \frac{2\left[\cos\left(\theta - \frac{\pi}{3}\right) + i \sin\left(\theta - \frac{\pi}{3}\right)\right]}{\cos(-\theta) + i \sin(-\theta)}$

$= 2\left(\cos\left(2\theta - \frac{\pi}{3}\right) + i \sin\left(2\theta - \frac{\pi}{3}\right)\right)$

, at $\theta = \frac{\pi}{6}$: $\therefore z = 2(\cos 0 + i \sin 0)$

The square roots of the number z

$=\sqrt{2}\left(\cos\frac{0+2\pi n}{2}+i\sin\frac{0+2\pi n}{2}\right)$

Where $n=0, 1$

, at $n=0$

$\therefore$ The first square root of the number z

$=\sqrt{2}\,(\cos 0^\circ + i\sin 0^\circ)$

, at $n=1$

$\therefore$ The second square root of the number z

$=\sqrt{2}\,(\cos\pi + i\sin\pi)$

[b] $x+3y-2z=0$, $x-8y+8z=0$

, $3x-2y+4z=0$

$$\begin{pmatrix}1 & 3 & -2\\ 1 & -8 & 8\\ 3 & -2 & 4\end{pmatrix}\begin{pmatrix}x\\ y\\ z\end{pmatrix}=\begin{pmatrix}0\\ 0\\ 0\end{pmatrix}$$

$$A=\begin{pmatrix}1 & 3 & -2\\ 1 & -8 & 8\\ 3 & -2 & 4\end{pmatrix} \quad \therefore |A|=0 , \begin{vmatrix}1 & 3\\ 1 & -8\end{vmatrix}\neq 0$$

$\therefore$ RK (A) = RK (A^*) = 2 < the number of variables.

$\therefore$ The equations have an infinite number of solutions one of them is the zero solution.

To find the form of solution we put $x=\ell$

By substituting in 1st equation

$\therefore \ell+3y-2z=0$ (1)

By substituting in 2nd equation :

$\therefore \ell-8y+8z=0$ (2)

from (1) , (2) : $\therefore 5\ell+4y=0$ $\quad\therefore y=\frac{-5}{4}\ell$

from (1) : $\therefore \ell+3\left(\frac{-5}{4}\ell\right)-2z=0$ $\quad\therefore z=\frac{-11}{8}\ell$

$\therefore$ The form of solution $=\left(\ell , \frac{-5}{4}\ell , \frac{-11}{8}\ell\right)$

5

[a] Let the general term is :

$T_{r+1}={}^{3n}C_r\,(x^2)^{3n-r}\left(\frac{1}{2x}\right)^r$

$={}^{3n}C_r\left(\frac{1}{2}\right)^r\times(x)^{6n-2r-r}$

$\therefore 6n-3r=0$ $\quad\therefore r=2n$

$\therefore$ The term free of x is T_{2n+1} , at $n=4$, $x=1$

$\therefore$ The term free of $x={}^{12}C_8\left(\frac{1}{2}\right)^8$

$\therefore$ The middle term $=T_7={}^{12}c_6\left(\frac{1}{2}\right)^6(x)^{6\times4-3\times6}$

$={}^{12}c_6\left(\frac{1}{2}\right)^6(1)$

$\therefore$ The ratio $=\frac{{}^{12}c_8}{{}^{12}c_6}\times\frac{\left(\frac{1}{2}\right)^8}{\left(\frac{1}{2}\right)^6}=\frac{15}{112}$

[b] $M_1\,(3 , 0 , 3)$, $r_1=4$

$M_2\,(-1 , 4 , k)$, $r_2=5$

$M_1M_2=\sqrt{16+16+(k-3)^2}=\sqrt{k^2-6k+41}$

In case of the two spheres touching then :

$M_1M_2=r_2+r_1$ or r_2-r_1

(1) Touching externally :

$k^2-6k+41=(5+4)^2$

$k^2-6k-40=0$ $\quad\therefore k=10$ or -4

(2) Touching internally :

$k^2-6k+41=(5-4)^2$

$k^2-6k+40=0$ has no solution in $\mathbb{R}$

$\therefore$ The two spheres can't be touching internally.

Model 9

1

(1) 10 (2) $\{2 , -2\}$ (3) $\frac{\sqrt{2}}{5}$

(4) 3 (5) $\frac{\sqrt{3}}{4}$ or $\frac{-\sqrt{3}}{4}$ (6) 9

2

(1) 0 (2) 3 (3) $6 , -\frac{2}{3}$

(4) $\pm 2\sqrt{3}$ (5) $-\frac{1}{2}$ (6) -1

3

[a] $z_1=2\left(\sin\frac{\pi}{3}+i\cos\frac{\pi}{3}\right)$

$=2\left(\cos\left(\frac{\pi}{2}-\frac{\pi}{3}\right)+i\sin\left(\frac{\pi}{2}-\frac{\pi}{3}\right)\right)$

$=2\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)$

, $z_2=\sqrt{2}\left(\sin\frac{\pi}{4}-i\cos\frac{\pi}{4}\right)$

$=\sqrt{2}\left(\cos\left(\frac{3\pi}{2}+\frac{\pi}{4}\right)+i\sin\left(\frac{3\pi}{2}+\frac{\pi}{4}\right)\right)$

$=\sqrt{2}\left(\cos\frac{7\pi}{4}+i\sin\frac{7\pi}{4}\right)$

$=\sqrt{2}\left(\cos\frac{-\pi}{4}+i\sin\frac{-\pi}{4}\right)$

$z_3=1+\sqrt{3}\,i$

$\therefore r=\sqrt{1^2+(\sqrt{3})^2}=2$

$\because x>0 , y>0$

$\therefore \theta$ lies in the first quadrant.

$\therefore \theta=\tan^{-1}\left(\frac{\sqrt{3}}{1}\right)=60^\circ=\frac{\pi}{3}$

$\therefore z_3=2\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)$

$$\therefore z=\frac{\left[2\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)\right]^3\times\left[\sqrt{2}\left(\cos\frac{-\pi}{4}+i\sin\frac{-\pi}{4}\right)\right]^4}{\left[2\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)\right]^5}$$

$$=\frac{8\left(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}\right)\times4\left(\cos(-\pi)+i\sin(-\pi)\right)}{32\left(\cos\frac{5\pi}{3}+i\sin\frac{5\pi}{3}\right)}$$

$=\cos\frac{-13\pi}{6}+i\sin\frac{-13\pi}{6}$

$=\cos\frac{-\pi}{6}+i\sin\frac{-\pi}{6}=e^{\frac{-\pi}{6}i}$

$\therefore$ The square roots of the number z

$$=\cos\left(\frac{\frac{-\pi}{6}+2\pi n}{2}\right)+i\sin\left(\frac{\frac{-\pi}{6}+2\pi n}{2}\right)$$

Where $n=0 , 1$

, at $n=0$

$\therefore$ The first square root of the number z

$=\cos\left(\frac{-\pi}{12}\right)+i\sin\left(\frac{-\pi}{12}\right)$

, at $n=1$

$\therefore$ The second square root of the number z

$=\cos\left(\frac{11\pi}{12}\right)+i\sin\left(\frac{11\pi}{12}\right)$

[b] $M_1=(-3 , 4 , 1)$, $M_2=(5 , -2 , 1)$

$\therefore$ Mid-point of $\overline{M_1M_2}=\left(\frac{5-3}{2} , \frac{4-2}{2} , \frac{1+1}{2}\right)$

$=(1 , 1 , 1)$

$\therefore 2a-3\ \ a+4a+6=0$ $\quad\therefore 3a=-6$

$\therefore a=-2$

4

[a] The matrix equation :

$$\begin{pmatrix}1 & -2 & 2\\ 3 & 0 & 4\\ 0 & -1 & 6\end{pmatrix}\begin{pmatrix}x\\ y\\ z\end{pmatrix}=\begin{pmatrix}2\\ 10\\ 5\end{pmatrix}$$

$$A=\begin{pmatrix}1 & -2 & 2\\ 3 & 0 & 4\\ 0 & -1 & 6\end{pmatrix} \quad \therefore |A|=34$$

$$\text{Adj}(A)=\begin{pmatrix}4 & 10 & -8\\ -18 & 6 & 2\\ -3 & 1 & 6\end{pmatrix}$$

$$A^{-1}=\frac{1}{34}\begin{pmatrix}4 & 10 & -8\\ -18 & 6 & 2\\ -3 & 1 & 6\end{pmatrix}$$

$$\therefore \begin{pmatrix}x\\ y\\ z\end{pmatrix}=\frac{1}{34}\begin{pmatrix}4 & 10 & -8\\ -18 & 6 & 2\\ -3 & 1 & 6\end{pmatrix}\begin{pmatrix}2\\ 10\\ 5\end{pmatrix}=\begin{pmatrix}2\\ 1\\ 1\end{pmatrix}$$

$\therefore x=2$, $y=1$, $z=1$

[b] $T_{r+1}={}^{5n}C_r\left(\frac{1}{x^3}\right)^r(x^2)^{5n-r}$

$={}^{5n}C_r\,(x)^{-3r}(x)^{10n-2r}$

$={}^{5n}C_r\,x^{10n-5r}$

, putting $10n-5r=0$ $\quad\therefore 2n=r$

$\therefore$ The value of the term free of x

$={}^{5n}C_{2n}=\frac{\lfloor 5n}{\lfloor 2n\,\lfloor 3n}$

5

[a] $\begin{pmatrix}k & 1 & 1\\ 1 & k & 1\\ 1 & 1 & k\end{pmatrix}\begin{pmatrix}x\\ y\\ z\end{pmatrix}=\begin{pmatrix}1\\ 1\\ 1\end{pmatrix}$

The equations have an infinite number of solutions when RK (A) = RK (A^*) < the number of variables.

Put $\begin{vmatrix}k & 1 & 1\\ 1 & k & 1\\ 1 & 1 & k\end{vmatrix}=0$

$\therefore k(k^2-1)-1(k-1)+1(1-k)=0$

$\therefore k(k^2-1)-k+1+1-k=0$

$\therefore k(k-1)(k+1)-2k+2=0$

$\therefore k(k-1)(k+1)-2(k-1)=0$

$\therefore (k-1)(k^2+k-2)=0$

$\therefore (k-1)^2(k+2)=0$

$\therefore k=1$ or $k=-2$

, at $k=1$ $\quad \therefore |A|=0 \quad \therefore 1 \le RK(A) < 3$

$$, A=\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

$$, \because A^* = \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{array}\right)$$

, $\because$ all the determinants of 2^{nd} degree = zero

$\therefore RK(A) = RK(A^*) = 1$

$\therefore$ There are an infinite number of solutions at $k=1$

, at $k=-2$ $\quad \therefore |A|=0 \quad \therefore 1 \le RK(A) < 3$

$$, A=\begin{pmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{pmatrix}$$

$$, \because \begin{vmatrix} -2 & 1 \\ 1 & -2 \end{vmatrix} = 3 \ne 0 \qquad \therefore RK(A)=2$$

$$, A^* = \left(\begin{array}{ccc|c} -2 & 1 & 1 & 1 \\ 1 & -2 & 1 & 1 \\ 1 & 1 & -2 & 1 \end{array}\right)$$

$$, \because \begin{vmatrix} 1 & 1 & 1 \\ -2 & 1 & 1 \\ 1 & -2 & 1 \end{vmatrix} = 9 \ne 0$$

$\therefore RK(A^*) = 3 \qquad , \because RK(A) \ne RK(A^*)$

$\therefore$ There are no solution at $k=-2$

[b] $d_1 (-3, 1, -2) - (-4, 1, 1) = (1, 0, -3)$

$d_2 = (1, \sqrt{5}, 2)$

$$\cos\theta = \frac{|(1,0,-3)\cdot(1,\sqrt{5},2)|}{\sqrt{1+0+9}\sqrt{1+5+4}} = \frac{|1-6|}{10} = \frac{1}{2}$$

$\therefore \theta = 60^\circ \qquad AB = \sqrt{10}$

$\therefore$ The distance from the point to the straight line

$= AB \sin\theta = \sqrt{10} \sin 60^\circ = \frac{\sqrt{30}}{2}$ length unit.

Model 10

1

(1) 4 (2) 5 (3) −4

(4) 60° (5) −18 (6) 2

2

(1) (c) (2) (b) (3) (d)

(4) (c) (5) (c) (6) (a)

3

[a] $13\,T_3 + 10\,T_4 + T_5 = 0$ (Dividing by T_4)

$$13 \frac{T_3}{T_4} + 10 + \frac{T_5}{T_4} = 0$$

$$13 \times \frac{3}{15-3+1} \times \frac{2x}{-3} + 10 + \frac{15-4+1}{4} \times \frac{-3}{2x} = 0$$

$$-2x + 10 + \frac{-9}{2x} = 0 \quad \text{(Multiplying by } (-2x))$$

$$4x^2 - 20x + 9 = 0 \qquad \therefore x = \frac{9}{2} \text{ or } x = \frac{1}{2}$$

$$\textbf{[b]}\ \text{L.H.S.} = \begin{vmatrix} y+z & x & x \\ y & z+x & y \\ z & z & x+y \end{vmatrix}$$

by doing $R_1 - (R_2 - R_3)$

$$= \begin{vmatrix} 0 & -2z & -2y \\ y & z+x & y \\ z & z & x+y \end{vmatrix}$$

$$= -2 \begin{vmatrix} 0 & z & y \\ y & z+x & y \\ z & z & x+y \end{vmatrix}$$

by doing $(R_2 - R_1)$, $(R_3 - R_1)$

$$= -2 \begin{vmatrix} 0 & z & y \\ y & x & 0 \\ z & 0 & x \end{vmatrix} \text{ exchange } R_2, R_3$$

$$= 2 \begin{vmatrix} 0 & z & y \\ z & 0 & x \\ y & x & 0 \end{vmatrix} = \text{R.H.S.}$$

4

[a] Putting $\varnothing = \frac{\pi}{2} - 2\theta$

$$\therefore \text{L.H.S} = \left(\frac{1 + \sin\left(\frac{\pi}{2} - 2\theta\right) + i\cos\left(\frac{\pi}{2} - 2\theta\right)}{1 + \sin\left(\frac{\pi}{2} - 2\theta\right) - i\cos\left(\frac{\pi}{2} - 2\theta\right)}\right)^n$$

$$= \left(\frac{1 + \cos 2\theta + i \sin 2\theta}{1 + \cos 2\theta - i\sin 2\theta}\right)^n$$

$$= \left(\frac{2\cos^2\theta + 2i\sin\theta\cos\theta}{2\cos^2\theta - 2i\sin\theta\cos\theta}\right)^n$$

$$= \left(\frac{2\cos\theta(\cos\theta + i\sin\theta)}{2\cos\theta(\cos\theta - i\sin\theta)}\right)^n$$

$$= \left(\frac{\cos\theta + i\sin\theta}{\cos(-\theta) + i\sin(-\theta)}\right)^n$$

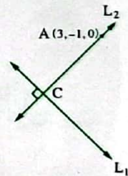
$$= (\cos 2\theta + i\sin 2\theta)^n \text{ but } 2\theta = \frac{\pi}{2} - \varnothing$$

$$= \left(\cos\left(\frac{\pi}{2} - \varnothing\right) + i\sin\left(\frac{\pi}{2} - \varnothing\right)\right)^n$$

$$= \cos n\left(\frac{\pi}{2} - \varnothing\right) + i \sin n\left(\frac{\pi}{2} - \varnothing\right)$$

[b] Let the two straight lines intersect at C

$C \in L_1$



$\therefore C = (2+t, 1+2t, 1-t)$

Direction vector of L_2

$, \vec{d_2} = \overrightarrow{AC}$

$= (-1+t, 2+2t, 1-t)$

, $\because$ the two straight lines are perpendicular

$\therefore \vec{d_1} \cdot \vec{d_2} = 0$

$\therefore (1, 2, -1) \cdot (-1+t, 2+2t, 1-t) = 0$

$\therefore -1 + t + 4 + 4t - 1 + t = 0$

$\therefore t = -\frac{1}{3}$

$\therefore \vec{d_2} = \left(\frac{-4}{3}, \frac{4}{3}, \frac{4}{3}\right) = (-1, 1, 1)$

$\therefore$ Equation of L_2

$, \vec{r} = (3, -1, 0) + t(-1, 1, 1)$

5

[a] Let $\frac{1}{x} = \ell$, $\frac{1}{y} = m$, $\frac{1}{z} = n$

$\therefore$ The equations becomes $, \ell + m + n = 1$

$, \ell - m + 2n = \frac{1}{2} \quad , \quad 2\ell + 3m - 4n = \frac{4}{3}$

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & -1 & 2 \\ 2 & 3 & -4 \end{pmatrix}, |A| = 11$$

$$A^{-1} = \frac{1}{11}\begin{pmatrix} -2 & 7 & 3 \\ 8 & -6 & -1 \\ 5 & -1 & -2 \end{pmatrix}$$

$$\begin{pmatrix} \ell \\ m \\ n \end{pmatrix} = \frac{1}{11}\begin{pmatrix} -2 & 7 & 3 \\ 8 & -6 & -1 \\ 5 & -1 & -2 \end{pmatrix} \times \begin{pmatrix} 1 \\ \frac{1}{2} \\ \frac{4}{3} \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ \frac{1}{3} \\ \frac{1}{6} \end{pmatrix}$$

$\ell = \frac{1}{2}$, then $x = 2$

$m = \frac{1}{3}$, then $y = 3$

$n = \frac{1}{6}$, then $z = 6$

[b] $\overrightarrow{AB} = \vec{B} - \vec{A} = (1, 0, \sqrt{3})$

$\vec{M} = (3, 2, 2\sqrt{3})$, $\|\vec{M}\| = 5$

The vector components of $\overrightarrow{AB}$ in direction of $\vec{M}$

$$= \frac{\overrightarrow{AB} \cdot \vec{M}}{\|\vec{M}\|^2}\vec{M} = \frac{3+0+6}{25}(3, 2, 2\sqrt{3})$$

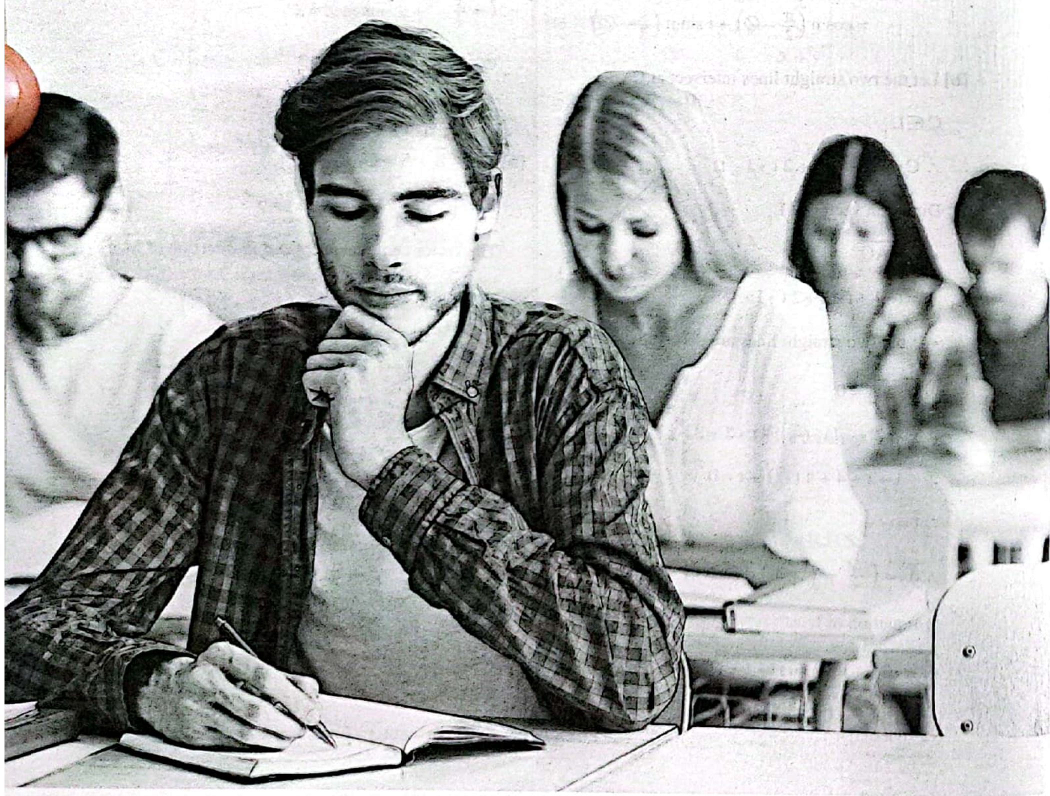
$$= \frac{9}{25}(3, 2, 2\sqrt{3})$$

$$= \left(\frac{27}{25}, \frac{18}{25}, \frac{18\sqrt{3}}{25}\right)$$

Answers

of

Egypt Exams



Answers of Egypt Exams

First session 2017

1 (c)

2 (b)

3

[a] $\because r=\sqrt{2}$, $\theta=\tan^{-1}\left(\frac{1}{1}\right)=\frac{\pi}{4}$

$\therefore z=\sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)$

$\therefore z^{\frac{1}{3}}=\sqrt[6]{2}\left(\cos\frac{\frac{\pi}{4}+2\pi n}{3}+i\sin\frac{\frac{\pi}{4}+2\pi n}{3}\right)$

where $n=0$, 1 , -1

, at $n=0$

$\therefore z_1=\sqrt[6]{2}\left(\cos\frac{\pi}{12}+i\sin\frac{\pi}{12}\right)=\sqrt[6]{2}\,e^{\frac{\pi}{12}i}$

, at $n=1$

$\therefore z_2=\sqrt[6]{2}\left(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}\right)=\sqrt[6]{2}\,e^{\frac{3\pi}{4}i}$

, at $n=-1$

$\therefore z_3=\sqrt[6]{2}\left(\cos\frac{-7\pi}{12}+i\sin\frac{-7\pi}{12}\right)=\sqrt[6]{2}\,e^{\frac{-7\pi}{12}i}$

[b] $\because r=\sqrt{(1)^2+\left(-\sqrt{3}\right)^2}=2$

, $\theta=\tan^{-1}\left(\frac{-\sqrt{3}}{1}\right)=\frac{-\pi}{3}$

$\therefore z=2\left(\cos\frac{-\pi}{3}+i\sin\frac{-\pi}{3}\right)$

$\therefore z^{\frac{3}{2}}=\left(\left[2\left(\cos\frac{-\pi}{3}+i\sin\frac{-\pi}{3}\right)\right]^3\right)^{\frac{1}{2}}$

$=\left(8\left(\cos-\pi+i\sin-\pi\right)\right)^{\frac{1}{2}}$

$=2\sqrt{2}\left(\cos\frac{-\pi+2\pi n}{2}+i\sin\frac{-\pi+2\pi n}{2}\right)$

where $n=0$, 1

, at $n=0$ $\quad\therefore z^{\frac{3}{2}}=2\sqrt{2}\left(\cos\frac{-\pi}{2}+i\sin\frac{-\pi}{2}\right)$

, at $n=1$ $\quad\therefore z^{\frac{3}{2}}=2\sqrt{2}\left(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}\right)$

4

by doing c_2-c_1 , c_3-c_1

$$\therefore \Delta=\begin{vmatrix}1 & 0 & 0\\ x & y-x & 0\\ x & 0 & -y-x\end{vmatrix}=1\times(y-x)\times(-y-x)$$

$$=-(y-x)(y+x)$$

$$=-(y^2-x^2)=x^2-y^2$$

5 (c)

6

The matrix equation is $\begin{pmatrix}2 & -3 & -1\\ 1 & 2 & 3\\ 1 & 0 & -2\end{pmatrix}\begin{pmatrix}x\\ y\\ z\end{pmatrix}=\begin{pmatrix}9\\ 15\\ 12\end{pmatrix}$

Put $A=\begin{pmatrix}2 & -3 & -1\\ 1 & 2 & 3\\ 1 & 0 & -2\end{pmatrix}$ $\quad\therefore |A|=-21$

$\text{Adj}(A)=\begin{pmatrix}-4 & -6 & -7\\ 5 & -3 & -7\\ -2 & -3 & 7\end{pmatrix}$

$\therefore A^{-1}=\frac{1}{-21}\begin{pmatrix}-4 & -6 & -7\\ 5 & -3 & -7\\ -2 & -3 & 7\end{pmatrix}$

$\therefore \begin{pmatrix}x\\ y\\ z\end{pmatrix}=A^{-1}\times\begin{pmatrix}9\\ 15\\ 12\end{pmatrix}$

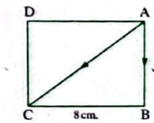
$=\frac{-1}{21}\begin{pmatrix}-4 & -6 & -7\\ 5 & -3 & -7\\ -2 & -3 & 7\end{pmatrix}\times\begin{pmatrix}9\\ 15\\ 12\end{pmatrix}=\begin{pmatrix}10\\ 4\\ -1\end{pmatrix}$

$\therefore x=10$, $y=4$, $z=-1$

7 (c) **8** (c) **9** (d)

10

[a] (1) $\overrightarrow{AB}\cdot\overrightarrow{AC}$



$=\|\overrightarrow{AB}\|\cdot\|\overrightarrow{AC}\|$

$\cos(\angle BAC)$

$=6\times10\times\frac{6}{10}=36$

(2) The component of $\overrightarrow{CD}$ in the direction of $\overrightarrow{BC}$

$=\frac{\overrightarrow{CD}\cdot\overrightarrow{BC}}{\|\overrightarrow{BC}\|}=\text{zero}$

(because they are perpendicular)

[b] $\therefore \cos^2\theta_1 + \cos^2\theta_2 + \cos^2\theta_3 = 1$

$, \because \theta_1 = \theta_2 = \theta_3 = \theta$

$\because 3\cos^2\theta = 1 \quad \therefore \cos^2\theta = \frac{1}{3}$

$\therefore \cos\theta = \pm\frac{1}{\sqrt{3}}$

$\therefore \vec{A} = 21\sqrt{3}\left(\pm\frac{1}{\sqrt{3}}\hat{i} \pm \frac{1}{\sqrt{3}}\hat{j} \pm \frac{1}{\sqrt{3}}\hat{k}\right)$

$= \pm 21(\hat{i} + \hat{j} + \hat{k})$

11 (b) **12** (c)

13

The normal direction vector of the plane :

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 6 & -2 & -1 \\ 1 & -3 & 3 \end{vmatrix} = (-9, -19, -16)$$

$, \because$ the plane contains the st. line

$\vec{r} = (0, 3, -5) + t_1 (6, -2, -1)$

$\therefore$ The plane contain the point $(0, 3, -5)$

$\therefore$ The equation is :

$\vec{r} \cdot (-9, -19, -16) = (0, 3, -5) \cdot (-9, -19, -16)$

$\therefore -9x - 19y - 16z - 23 = 0$

14

$\because$ The equation is : $\frac{x}{4} + \frac{y}{6} + \frac{z}{3} = 1$

$\therefore$ The points are $A(4, 0, 0)$, $B(0, 6, 0)$, $C(0, 0, 3)$

$, \overrightarrow{AB} = \vec{B} - \vec{A} = (-4, 6, 0)$

$, \overrightarrow{AC} = \vec{C} - \vec{A} = (-4, 0, 3)$

$$\therefore \overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -4 & 6 & 0 \\ -4 & 0 & 3 \end{vmatrix}$$

$= 18\hat{i} + 12\hat{j} + 24\hat{k}$

$\therefore$ Area of the triangle $= \frac{1}{2}\|\overrightarrow{AB} \times \overrightarrow{AC}\|$

$= \frac{1}{2}\sqrt{(18)^2 + (12)^2 + (24)^2}$

$= 3\sqrt{29}$ square unit.

15 (c) **16** (b) **17** (c)

18

$\because T_3 = {}^nC_2\, x^2 = 17 \qquad (1)$

$, \because 3\, T_2 \times T_4 = 544$

$\therefore 3\, {}^nC_1\, x \times {}^nC_3\, x^3 = 544 \qquad (2)$

Divide (2) by square of (1)

$\therefore \frac{3n \times {}^nC_3}{({}^nC_2)^2} = \frac{544}{(17)^2}$

$\therefore \frac{3n \times n(n-1)(n-2) \times (\lfloor 2)^2}{\lfloor 3 \times n^2 (n-1)^2} = \frac{32}{17}$

$\therefore \frac{n-2}{n-1} = \frac{16}{17} \quad \therefore 17n - 34 = 16n - 16$

$\therefore n = 18$

by substituting in (1) :

$\therefore {}^{18}C_2\, x^2 = 17 \quad \therefore x^2 = \frac{1}{9} \quad \therefore x = \pm\frac{1}{3}$

19 (b)

Second session 2017

1 (c)

2

$\because$ The required plane is parallel to the plane

$2x + y - 4z = 0$

, then the equation the required plane in the form

$2x + y - 4z + k = 0$

$\because$ The plane lies at a distance $\sqrt{21}$ from the point $(1, 2, 0)$

$\therefore \sqrt{21} = \frac{|2(1) + (2) - 4(0) + k|}{\sqrt{2^2 + 1^2 + (-4)^2}}$

$\sqrt{21} = \frac{|4 + k|}{\sqrt{21}}$

$\therefore |4 + k| = 21 \quad \therefore 4 + k = \pm 21$

$\therefore 4 + k = 21$, then $k = 17$

or $4 + k = -21$, then $k = -25$

3

$$A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 1 & 1 & 0 \end{pmatrix}$$

$\therefore |A| = -1 \neq 0$

$\therefore RK(A) = 3$

$\therefore$ The system of the equations has unique solution

$$\therefore \text{Matrix of cofactors} = \begin{pmatrix} 1 & -1 & -1 \\ 2 & -2 & -1 \\ -2 & 1 & 1 \end{pmatrix}$$

$$\text{Adj}(A) = \begin{pmatrix} 1 & 2 & -2 \\ -1 & -2 & 1 \\ -1 & -1 & 1 \end{pmatrix}$$

$$A^{-1} = \frac{1}{|A|} \times \text{adj}(A) = \begin{pmatrix} -1 & -2 & 2 \\ 1 & 2 & -1 \\ 1 & 1 & -1 \end{pmatrix}$$

$$\therefore \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -1 & -2 & 2 \\ 1 & 2 & -1 \\ 1 & 1 & -1 \end{pmatrix}\begin{pmatrix} 5 \\ 1 \\ 7 \end{pmatrix} = \begin{pmatrix} 7 \\ 0 \\ -1 \end{pmatrix}$$

$x = 7 \;,\; y = 0 \;,\; z = -1$

4 (b) **5** (b) **6** (b)

7

[a] $z^3 = -8i = 8e^{\frac{-\pi}{2}i}$

$\therefore z = 8^{\frac{1}{3}} e^{\frac{\frac{-\pi}{2} + 2\pi k}{3} i} \;, k = 0, 1, -1$

1^{st} root $= 2e^{\frac{-\pi}{6}i} = 2\left(\cos\frac{-\pi}{6} + i\sin\frac{-\pi}{6}\right)$

2^{nd} root $= 2e^{\frac{\pi}{2}i} = 2\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$

3^{rd} root $= 2e^{\frac{-5\pi}{6}i} = 2\left(\cos\frac{-5\pi}{6} + i\sin\frac{-5\pi}{6}\right)$

[b] $\because z = \frac{1}{\sqrt{2}}(1 + i) = \left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

$\therefore z^{\frac{1}{2}} = 1\left(\cos\left(\frac{\frac{\pi}{4} + 2\pi k}{2}\right) + i\sin\left(\frac{\frac{\pi}{4} + 2\pi k}{2}\right)\right)$

$, k = 0, -1$

1^{st} root $= \left(\cos\frac{\pi}{8} + i\sin\frac{\pi}{8}\right)$

2^{nd} root $= \left(\cos\left(\frac{-7}{8}\pi\right) + i\sin\left(\frac{-7}{8}\pi\right)\right)$

8 (a) **9** (a) **10** (d)

11

$T_{r+1} = {}^{11}C_r \left(\frac{2}{x^2}\right)^r (x^2)^{11-r}$

$= ({}^{11}C_r \times 2^r) \times x^{-2r+22-2r}$

$= {}^{11}C_r\, 2^r \times x^{22-4r}$

Let $22 - 4r = 0$

$\therefore r = \frac{22}{4} \notin \mathbb{N}$

$\therefore$ There is no exist free term of x in this expansion.

12

Area of parallelogram $= \|\vec{A} \times \vec{B}\|$

$$\therefore \vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 6 & 3 \\ -6 & -2 & -4 \end{vmatrix}$$

$= -18\hat{i} - 6\hat{j} + 30\hat{k}$

$\therefore \|\vec{A} \times \vec{B}\| = 6\sqrt{35}$ square unit.

13 (d) **14** (b) **15** (b)

16

$$\begin{vmatrix} 3x & 3x & 3x \\ 1 & b & a \\ a+b & a+1 & b+1 \end{vmatrix}$$

$$= 3x \begin{vmatrix} 1 & 1 & 1 \\ 1 & b & a \\ a+b & a+1 & b+1 \end{vmatrix}$$

by doing $R_3 + R_2$

$$= 3x \begin{vmatrix} 1 & 1 & 1 \\ 1 & b & a \\ a+b+1 & a+b+1 & a+b+1 \end{vmatrix}$$

$$= 3x(a+b+1) \begin{vmatrix} 1 & 1 & 1 \\ 1 & b & a \\ 1 & 1 & 1 \end{vmatrix}$$

$$= 3x(a+b+1) \times 0 = 0$$

(because $R_1 = R_3$)

17 (d)

18

[a] $\because L_1 \parallel L_2$

$\therefore \overrightarrow{d_1} = k\,\overrightarrow{d_2}$

$\therefore (2, 3, a) = k(b, 6, 2)$

$\therefore \frac{2}{b} = \frac{3}{6} = \frac{a}{2}$

$\therefore b = 4 \quad , \quad a = 1$

[b] $\overrightarrow{d_1} = (4, -2, 2)$, $\overrightarrow{d_2} = (-6, 21, 33)$

$\therefore \overrightarrow{d_1} \cdot \overrightarrow{d_2} = (4, -2, 2) \cdot (-6, 21, 33)$

$= -24 - 42 + 66 = \text{zero}$

$\therefore \overrightarrow{d_1} \perp \overrightarrow{d_2}$

$\therefore$ The two straight lines are perpendicular.

19 (a)

First session 2018

1 (b) **2** (b) **3** (d)

4

[a] $z = \frac{8(\sqrt{3}+i)}{\sqrt{3}-i} \times \frac{\sqrt{3}+i}{\sqrt{3}+i} = \frac{8(2+2\sqrt{3}\,i)}{4} = 4 + 4\sqrt{3}\,i$

$\therefore r = \sqrt{4^2 + (4\sqrt{3})^2} = 8$

$, \theta = \frac{\pi}{3}$

$\therefore z = 8\left(\cos\frac{\pi}{3} + i \sin\frac{\pi}{3}\right) = 8\,e^{\frac{\pi}{3}i}$

$\therefore$ the cubic roots of the number z is

$\sqrt[3]{z} = 2\,e^{\frac{\frac{\pi}{3}+2\pi n}{3}i}$ where $n = 0, 1, -1$

at $n = 0$ $\quad \therefore z_1 = 2\,e^{\frac{\pi}{9}i}$

at $n = 1$ $\quad \therefore z_2 = 2\,e^{\frac{7\pi}{9}i}$

at $n = -1$ $\quad \therefore z_3 = 2\,e^{\frac{-5}{9}\pi i}$

[b] $(x + iy)(1 - 3i) = 37\left(\frac{1}{3-4\omega^2} + \frac{1}{7+4\omega^2}\right)$

$= 37\left(\frac{7+4\omega^2+3-4\omega^2}{(3-4\omega^2)(7+4\omega^2)}\right)$

$= 37\left(\frac{10}{21+12\omega^2-28\omega^2-16\omega^4}\right)$

$= \frac{37 \times 10}{(21-16\omega-16\omega^2)} = \frac{370}{21+16}$

$= 10$

$\therefore x + iy = \frac{10}{1-3i} \times \frac{1+3i}{1+3i} = \frac{10(1+3i)}{1+9} = 1 + 3i$

$\therefore x = 1 \quad , \quad y = 3$

5 (b) **6** (c) **7** (b)

8

[a] (1) $\overrightarrow{BA} = \overrightarrow{A} - \overrightarrow{B} = (-1, -2, -3)$

$, \overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC} = (1, 2, 3) + (-1, 4, 0)$

$= (0, 6, 3)$

$, \cos(\angle ABC) = \frac{\overrightarrow{BA} \cdot \overrightarrow{BC}}{\|\overrightarrow{BA}\| \|\overrightarrow{BC}\|}$

$= \frac{(-1, -2, -3) \cdot (-1, 4, 0)}{\sqrt{(-1)^2+(-2)^2+(-3)^2} \times \sqrt{(-1)^2+(4)^2+0^2}}$

$= \frac{-7}{\sqrt{17}\,\sqrt{14}}$

$\therefore m(\angle ABC) \simeq 116° \, 59' \simeq 117°$

(2) The vector component of $\overrightarrow{AC}$ in direction of $\overrightarrow{AB}$

$= \left(\frac{\overrightarrow{AC} \cdot \overrightarrow{AB}}{\|\overrightarrow{AB}\|^2}\right)\overrightarrow{AB}$

$= \left(\frac{(0, 6, 3) \cdot (1, 2, 3)}{(1)^2 + (2)^2 + (3)^2}\right)(1, 2, 3)$

$= \frac{21}{14}(1, 2, 3) = \left(\frac{3}{2}, 3, \frac{9}{2}\right)$

[b] (1) $\because \overrightarrow{A} \cdot (\overrightarrow{B} \times \overrightarrow{C}) = \begin{vmatrix} 1 & 4 & 2 \\ -3 & 2 & 1 \\ -1 & 1 & 4 \end{vmatrix}$

$= 7 - 4(-11) + 2(-1) = 49$

$\therefore$ Volume of parallelpiped $= |49| = 49$ cubic unit.

(2) $\overrightarrow{A} \times \overrightarrow{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 4 & 2 \\ -3 & 2 & 1 \end{vmatrix} = -7\hat{j} + 14\hat{k}$

$\therefore \|\overrightarrow{A} \times \overrightarrow{B}\| = \sqrt{(-7)^2 + (14)^2} = 7\sqrt{5}$

$\because$ The area of the base determined by the two vectors $\overrightarrow{A}, \overrightarrow{B} = 7\sqrt{5}$ square unit.

$\therefore$ The height $= \frac{\text{volume}}{\text{base area}} = \frac{49}{7\sqrt{5}}$

$= \frac{7\sqrt{5}}{5} \simeq 3.13$ length unit.

9 (b) **10** (a) **11** (b)

12

L.H.S. $= \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix}$

[by doing $(C_2 - C_1), (C_3 - C_1)$]

$$= \begin{vmatrix} 1 & 0 & 0 \\ a & b-a & c-a \\ a^2 & b^2-a^2 & c^2-a^2 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 0 & 0 \\ a & b-a & c-a \\ a^2 & (b-a)(b+a) & (c-a)(c+a) \end{vmatrix}$$

by taking $(b-a), (c-a)$ common factor.

$$= (b-a)(c-a) \begin{vmatrix} 1 & 0 & 0 \\ a & 1 & 1 \\ a^2 & b+a & c+a \end{vmatrix}$$

[by doing $(C_3 - C_2)$]

$$= (b-a)(c-a)\begin{vmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ a^2 & b+a & c-b \end{vmatrix}$$

$$= (b-a)(c-a)(c-b) = \text{R.H.S.}$$

13 (d) **14** (c)

15

$$T_{r+1} = {}^{12}C_r \left(-\frac{1}{x^2}\right)^r (x^2)^{12-r} = {}^{12}C_r \times (-1)^r \times x^{24-4r}$$

Put $24 - 4r = 4$ $\therefore r = 5$

$\therefore$ The term contains x^4 is $T_6 = {}^{12}C_5 (-1)^5 x^4 = -792x^4$

$\because$ The middle term is $T_7 = {}^{12}C_6 \left(\frac{-1}{x^2}\right)^6 \times (x^2)^6 = 924$

$$\therefore \frac{\text{coeff. of } T_6}{\text{middle term}} = \frac{-792}{924} = \frac{-6}{7}$$

16

$\vec{n} \cdot \vec{r} = \vec{n} \cdot \vec{A}$ (1)

$(4, 10, -7) \cdot \vec{r} = (4, 10, -7) \cdot (2, -1, 0)$

$\therefore (4, 10, -7) \cdot \vec{r} = -2$ (vector form)

$, 4(x-2) + 10(y+1) - 7z = 0$ (standard form)

$, 4x + 10y - 7z + 2 = 0$ (general form)

17 (b)

18

$\because$ The straight line makes equals angles with positive directions of Cartesian axes.

$\therefore \cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$

$\therefore \cos\theta_x = \cos\theta_y = \cos\theta_z = \pm\frac{1}{\sqrt{3}}$

$\therefore$ Direction vector of the straight line is $\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$

i.e. $\vec{d} = (1, 1, 1)$

$\therefore$ Vector form is : $\vec{r} = (3, 2, -1) + t(1, 1, 1)$

$\therefore$ Parametric form : $x = 3 + t$, $y = 2 + t$, $z = -1 + t$

$\therefore$ Cartesian form : $x - 3 = y - 2 = z + 1$

19

The matrix equation : $\begin{pmatrix} 0 & -3 & 2 \\ 5 & 1 & 0 \\ 1 & -2 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 7 \\ 4 \\ 1 \end{pmatrix}$

$$\therefore A = \begin{pmatrix} 0 & -3 & 2 \\ 5 & 1 & 0 \\ 1 & -2 & -1 \end{pmatrix}, |A| = -37$$

$$A^{adj} = \begin{pmatrix} -1 & 5 & -11 \\ -7 & -2 & -3 \\ -2 & 10 & 15 \end{pmatrix}^t = \begin{pmatrix} -1 & -7 & -2 \\ 5 & -2 & 10 \\ -11 & -3 & 15 \end{pmatrix}$$

$$\therefore A^{-1} = \frac{1}{-37}\begin{pmatrix} -1 & -7 & -2 \\ 5 & -2 & 10 \\ -11 & -3 & 15 \end{pmatrix}$$

$$\therefore \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{-1}{37}\begin{pmatrix} -1 & -7 & -2 \\ 5 & -2 & 10 \\ -11 & -3 & 15 \end{pmatrix}\begin{pmatrix} 7 \\ 4 \\ 1 \end{pmatrix}$$

$$= \frac{-1}{37}\begin{pmatrix} -37 \\ 37 \\ -74 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$$

$\therefore x = 1$, $y = -1$, $z = 2$

Second session 2018

1 (c) **2** (a) **3** (d)

4

$$\text{L.H.S.} = \begin{vmatrix} a & b & c \\ b & a & c \\ b & c & a \end{vmatrix} \left(\text{by doing } (C_1 + C_2 + C_3)\right)$$

(and taking $(a+b+c)$ a common factor from C_1)

$$= (a+b+c)\begin{vmatrix} 1 & b & c \\ 1 & a & c \\ 1 & c & a \end{vmatrix}$$

(by doing $(R_2 - R_1)$, $(R_3 - R_1)$)

$$= (a+b+c)\begin{vmatrix} 1 & b & c \\ 0 & a-b & 0 \\ 0 & c-b & a-c \end{vmatrix}$$

by interchanging R_2 , R_3

$$= -(a+b+c)\begin{vmatrix} 1 & b & c \\ 0 & c-b & a-c \\ 0 & a-b & 0 \end{vmatrix}$$

(by interchanging C_2 with C_3)

$$= (a+b+c)\begin{vmatrix} 1 & c & b \\ 0 & a-c & c-b \\ 0 & 0 & a-b \end{vmatrix}$$

$$= (a-b)(a-c)(a+b+c) = \text{R.H.S.}$$

5 (c) **6** (d)

7

$\because T_4 = 7$

$\therefore {}^8C_3 x^3 = 7$ $\therefore x^3 = \frac{1}{8}$ $\therefore x = \frac{1}{2}$

The middle term in the expansion is T_5

$$\therefore \frac{T_6}{T_5} = \frac{8-5+1}{5} \times \frac{x}{1}$$

$$= \frac{4}{5} \times \frac{1}{2} = \frac{2}{5}$$

8

The direction vector of the straight line = $(2, 4, 3)$

$\therefore \vec{r} = (-2, 3, 5) + t(2, 4, 3)$ (The vector form)

$, x = -2 + 2t$, $y = 3 + 4t$, $z = 5 + 3t$

(The parametric form)

$$, \frac{x+2}{2} = \frac{y-3}{4} = \frac{z-5}{3}$$

(The cartesian form)

9 (a)

10

$(2, -3, 4) \cdot \vec{r} = (2, -3, 4) \cdot (1, -1, 4)$

$\therefore (2, -3, 4) \cdot \vec{r} = 21$

(The vector form)

$, 2(x-1) - 3(y+1) + 4(z-4)$

(The standard form)

$, 2x - 3y + 4z - 21 = 0$

(The general form)

11

The matrix equation : $\begin{pmatrix} 1 & -2 & 0 \\ -1 & 1 & 2 \\ 1 & 0 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \\ -1 \end{pmatrix}$

$$\therefore A = \begin{pmatrix} 1 & -2 & 0 \\ -1 & 1 & 2 \\ 1 & 0 & -2 \end{pmatrix} , |A| = -2$$

$$, \text{Adj}(A) = \begin{pmatrix} -2 & 0 & -1 \\ -4 & -2 & -2 \\ -4 & -2 & -1 \end{pmatrix}^t$$

$$\therefore A^{-1} = \frac{1}{|A|}\begin{pmatrix} -2 & -4 & -4 \\ 0 & -2 & -2 \\ -1 & -2 & -1 \end{pmatrix}$$

$$\therefore \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{-2}\begin{pmatrix} -2 & -4 & -4 \\ 0 & -2 & -2 \\ -1 & -2 & -1 \end{pmatrix}\begin{pmatrix} 5 \\ 0 \\ -1 \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$$

$\therefore x = 3$, $y = -1$, $z = 2$

12 (b) **13** (b) **14** (d)

15

[a] $z=\frac{16}{1-\sqrt{3}\,i}\times\frac{1+\sqrt{3}\,i}{1+\sqrt{3}\,i}=4+4\sqrt{3}\,i$

$,\ r=\sqrt{(4)^2+\left(4\sqrt{3}\right)^2}=8\ ,\ x>0\ ,\ y>0$

$\therefore$ θ lies in the 1st quadrant

$\therefore \theta=60°=\frac{\pi}{3}$

$\therefore z=8\left[\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right]$

$\therefore \sqrt[3]{z}=z\left[\cos\frac{\frac{\pi}{3}+2\pi n}{3}+i\sin\frac{\frac{\pi}{3}+2\pi n}{3}\right]$

where $n=0\ ,1\ ,-1$

at $n=0$

The first cubic root $=2\left[\cos\frac{\pi}{9}+i\sin\frac{\pi}{9}\right]$

at $n=1$

The second cubic root $=2\left[\cos\frac{7\pi}{9}+i\sin\frac{7\pi}{9}\right]$

at $n=-1$

The third cubic root $=2\left[\cos\frac{-5\pi}{9}+i\sin\frac{-5\pi}{9}\right]$

[b] $\frac{1+10\,(\omega+\omega^2)}{1-3\,(\omega+\omega^2)}=(Ki)^2$

$\therefore \frac{1-10}{1+3}=K^2 i^2$

$\therefore \frac{-9}{4}=-K^2 \qquad \therefore K^2=\frac{9}{4}$

$\therefore K=\pm\frac{3}{2}$

16 (c) **17** (b) **18** (d)

19

[a] $\overrightarrow{AB}=\overrightarrow{B}-\overrightarrow{A}=(2\ ,3\ ,1)$

$,\ \overrightarrow{AC}=\overrightarrow{C}-\overrightarrow{A}=(m-1\ ,-1\ ,3-10\,m)$

(1) $\because$ A , B , C are collinear

$\therefore \overrightarrow{AB}\ //\ \overrightarrow{AC}$

$\therefore \frac{m-1}{2}=\frac{-1}{3}=\frac{3-10\,m}{1}$

$\therefore m-1=\frac{-2}{3}$

$\therefore m=\frac{1}{3}$

(2) $\because \overrightarrow{AB}\perp\overrightarrow{AC}$

$\therefore \overrightarrow{AB}\cdot\overrightarrow{AC}=(2\ ,3\ ,1)\cdot(m-1\ ,-1\ ,3-10\,m)=0$

$\therefore 2\,m-2+(-3)+3-10\,m=0$

$\therefore -8\,m=2 \qquad \therefore m=\frac{-1}{4}$

[b] (1) $\overrightarrow{AB}=\overrightarrow{B}-\overrightarrow{A}=(3\ ,2\ ,3)$

$,\ \overrightarrow{BC}=\overrightarrow{C}-\overrightarrow{B}=(-2\ ,2\ ,0)$

$,\ \overrightarrow{DC}=\overrightarrow{C}-\overrightarrow{D}=(3\ ,2\ ,3)$

$,\ \overrightarrow{AD}=\overrightarrow{D}-\overrightarrow{A}=(-2\ ,2\ ,0)$

$\therefore \overrightarrow{AB}=\overrightarrow{DC}\ ,\ \overrightarrow{BC}=\overrightarrow{AD}$

$\therefore$ ABCD is a parallelogram

$\therefore \overrightarrow{AB}\times\overrightarrow{AD}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\3&2&3\\-2&2&0\end{vmatrix}$

$=-6\,\hat{i}+6\,\hat{j}+10\,\hat{k}$

$\therefore$ The area of the parallelogram $=\|\overrightarrow{AB}\times\overrightarrow{AD}\|$

$=\sqrt{(-6)^2+(-6)^2+(10)^2}$

$=2\sqrt{43}$ area unit.

(2) The unit vector perpendicular to the plane of the parallelogram

$=\frac{\overrightarrow{AB}\times\overrightarrow{AD}}{\|\overrightarrow{AB}\times\overrightarrow{AD}\|}$

$=\left(\frac{-6}{2\sqrt{43}}\ ,\frac{-6}{2\sqrt{43}}\ ,\frac{10}{2\sqrt{43}}\right)$

$=\left(\frac{-3}{\sqrt{43}}\ ,\frac{-3}{\sqrt{43}}\ ,\frac{5}{\sqrt{43}}\right)$

First session 2019

1 (d) **2** (d)

3

$\begin{vmatrix}1&x&x\\x&1&x\\x&x&1\end{vmatrix}=0$ (by doing (C_3-C_2))

$\therefore \begin{vmatrix}1&x&0\\x&1&x-1\\x&x&1-x\end{vmatrix}=0$ (by doing (R_2+R_3))

$\therefore \begin{vmatrix}1&x&0\\2x&x+1&0\\x&x&1-x\end{vmatrix}=0$ (by doing $(C_2-x\,C_1)$)

$\therefore \begin{vmatrix}1&0&0\\2x&x+1-2x^2&0\\x&x-x^2&1-x\end{vmatrix}=0$

$\therefore 1\times(1+x-2\,x^2)\,(1-x)=0$

$\therefore (1-x)\,(1+2\,x)\,(1-x)=0$

$\therefore x=1$ or $x=\frac{-1}{2}$

Another solution :

$\begin{vmatrix}1&x&x\\x&1&x\\x&x&1\end{vmatrix}=0$ (by doing $C_1+C_2+C_3$)

$\therefore \begin{vmatrix}2x+1&x&x\\2x+1&1&x\\2x+1&x&1\end{vmatrix}=0$

$(2\,x+1)\begin{vmatrix}1&x&x\\1&1&x\\1&x&1\end{vmatrix}=0$

(by doing (R_2-R_1) , (R_3-R_1))

$\therefore (2\,x+1)\begin{vmatrix}1&x&x\\0&1-x&0\\0&0&1-x\end{vmatrix}=0$

$\therefore (2\,x+1)\,(1-x)^2=0$

$\therefore x=-\frac{1}{2}$ or $x=1$

4

$\because \frac{4}{1}\neq\frac{1}{-1}\neq\frac{3}{2}$

$\therefore$ The two straight lines are not parallel

, at the point of intersection : $\overrightarrow{r_1}=\overrightarrow{r_2}$

$\therefore (3\ ,-1\ ,2)+t_1\,(4\ ,1\ ,3)$

$=(0\ ,4\ ,-1)+t_2\,(1\ ,-1\ ,2)$

$\therefore 3+4\,t_1=t_2 \qquad \therefore 4\,t_1-t_2=-3$ (1)

$,\ -1+t_1=4-t_2 \qquad \therefore t_1+t_2=5$ (2)

$,\ 2+3\,t_1=-1+2\,t_2 \qquad \therefore 3\,t_1-2\,t_2=-3$ (3)

From (1) , (2) we get : $t_1=\frac{2}{5}\ ,t_2=\frac{23}{5}$

by substitute in equation (3) :

$3\times\frac{2}{5}-2\times\frac{23}{5}\neq-3$

i.e. These values don't satisfy equation (3)

$\therefore$ The two straight lines are not intersecting.

$\therefore$ The two straight lines are skew.

5 (c) **6** (a) **7** (a)

8

[a] $z=\left(\frac{8}{1+\sqrt{3}\,i}\times\frac{1-\sqrt{3}\,i}{1-\sqrt{3}\,i}\right)=2-2\sqrt{3}\,i$

$\therefore r=4 \qquad ,\because x>0\ ,\ y<0$

$\therefore$ θ lies in the fourth quadrant.

$\therefore \theta=\tan^{-1}\left(\frac{-2\sqrt{3}}{2}\right)=-60°=\frac{-\pi}{3}$

$\therefore z=4\left(\cos\left(\frac{-\pi}{3}\right)+i\sin\left(\frac{-\pi}{3}\right)\right)=4\,e^{\frac{-\pi}{3}i}$

, the square roots of the number $z=2\,e^{\left(\frac{\frac{-\pi}{3}+2\pi n}{2}\right)i}$

where $n=0\ ,1$

at $n=0$: $\therefore$ The first square root $=2\,e^{\frac{-\pi}{6}i}$

at $n=1$: $\therefore$ The second square root $=2\,e^{\frac{5\pi}{6}i}$

[b] $(x-1)^6-9\,(x-1)^3+8=0$

$[(x-1)^3-8]\,[(x-1)^3-1]=0$

In case of

$(x-1)^3=8$

, $\because$ the cubic roots of the number 8 are $2\ ,2\,\omega\ ,2\,\omega^2$

$\therefore x-1=2$ hence $x=3$

or $x-1=2\,\omega$ hence $x=2\,\omega+1$

or $x-1=2\,\omega^2$ hence $x=2\,\omega^2+1$

In case of $(x-1)^3=1$

, ∵ the cubic roots of the number 1 are $1, \omega, \omega^2$

$\therefore x - 1 = 1$ hence $x = 2$

or $x - 1 = \omega$ hence $x = 1 + \omega = -\omega^2$

or $x - 1 = \omega^2$ hence $x = 1 + \omega^2 = -\omega$

9 ⓑ

10

$\because \frac{x}{1} + \frac{y}{2} + \frac{z}{3} = 1$

$\therefore 6x + 3y + 2z = 6$ (General form)

$\because$ The plane passes through $(1, 0, 0)$

$\therefore (6, 3, 2) \cdot \vec{r} = (6, 3, 2) \cdot (1, 0, 0)$

$\therefore (6, 3, 2) \cdot \vec{r} = 6$ (Vector form)

$\therefore 6(x - 1) + 3y + 2z = 0$ (Standard form)

11

$$\begin{pmatrix} 2 & -4 & -9 \\ -1 & 2 & 3 \\ -3 & 6 & 9 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$$

$$A = \begin{pmatrix} 2 & -4 & -9 \\ -1 & 2 & 3 \\ -3 & 6 & 9 \end{pmatrix} \qquad \therefore |A| = 0$$

$$, \because \begin{vmatrix} -4 & -9 \\ 2 & 3 \end{vmatrix} \neq 0 \qquad \therefore RK(A) = 2$$

$$, \because A^* = \left(\begin{array}{ccc|c} 2 & -4 & -9 & 1 \\ -1 & 2 & 3 & 0 \\ -3 & 6 & 9 & -1 \end{array}\right)$$

$$, \because \begin{vmatrix} -4 & -9 & 1 \\ 2 & 3 & 0 \\ 6 & 9 & -1 \end{vmatrix} = -6 \neq 0$$

$\therefore RK(A^*) = 3 \qquad \therefore RK(A) \neq RK(A^*)$

$\therefore$ The equations have no solution.

12 ⓐ **13** ⓓ **14** ⓒ

15

$T_{r+1} = {}^{15}C_r (x^2)^r \left(\frac{1}{x}\right)^{15-r} = {}^{15}C_r (x)^{-15+3r}$

Put $-15 + 3r = 0 \qquad \therefore r = 5$

i.e. T_6 is the term free of x

$\therefore$ The value of the term free of x is ${}^{15}C_5 = 3003$

, $\because$ The two middle terms are T_8, T_9

$\therefore T_8 = T_9 \qquad \therefore \frac{T_9}{T_8} = \frac{15-8+1}{8} \times x^3 = 1$

$\therefore x^3 = 1 \qquad \therefore x = 1$

16 ⓐ **17** ⓐ **18** ⓑ

19

[a] $\overrightarrow{AB} = \vec{B} - \vec{A} = (1, 0, -1)$

, $\overrightarrow{BC} = \vec{C} - \vec{B} = (-1, 1, 0)$

$$\therefore \overrightarrow{AB} \times \overrightarrow{BC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{vmatrix} = \hat{i} + \hat{j} + \hat{k}$$

$\therefore \|\overrightarrow{AB} \times \overrightarrow{BC}\| = \sqrt{3}$

$$\therefore \vec{u} = \pm \frac{\overrightarrow{AB} \times \overrightarrow{BC}}{\|\overrightarrow{AB} \times \overrightarrow{BC}\|} = \pm\left(\frac{1}{\sqrt{3}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \frac{1}{\sqrt{3}}\hat{k}\right)$$

Another solution :

$\because$ Equation of the plane passing through the points A, B, C is $x + y + z = 1$

$\therefore$ Perpendicular direction vector $= (1, 1, 1)$

$\therefore$ The unit vector perpendicular to the plane

$$= \pm \frac{(1, 1, 1)}{\sqrt{1^2 + 1^2 + 1^2}} = \pm\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$$

[b] $M_1 = (-1, 4, k)$, $r_1 = 5$ length unit.

$M_2 = (3, 0, 3)$, $r_2 = 4$ length unit.

$\therefore M_1M_2 = \sqrt{(-4)^2 + (4)^2 + (k-3)^2}$

$\because$ The two circles are externally tangential.

$\therefore M_1M_2 = r_1 + r_2$

$\therefore \sqrt{32 + (k-3)^2} = 9$

$\therefore (k-3)^2 = 49$

$k - 3 = \pm 7$

$\therefore k = 10$ or $k = -4$

Second session 2019

1 ⓒ **2** ⓒ **3** ⓓ

4

[a] $\overrightarrow{AB} = \vec{B} - \vec{A} = (1, 0, \sqrt{3})$

, $\vec{M} = (3, 2, 2\sqrt{3})$, $\|\vec{M}\| = 5$

The vector components of $\overrightarrow{AB}$ in direction of $\vec{M}$

$$= \left(\frac{\overrightarrow{AB} \cdot \vec{M}}{\|\vec{M}\|^2}\right)\vec{M} = \frac{3+0+6}{25}(3, 2, 2\sqrt{3})$$

$$= \frac{9}{25}(3, 2, 2\sqrt{3})$$

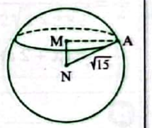
$$= \left(\frac{27}{25}, \frac{18}{25}, \frac{18\sqrt{3}}{25}\right)$$

[b] Centre of the sphere is

$N(-3, -2, 1)$

, length of its radius

$r = \sqrt{15}$ length unit.



, Let M be the centre of the resulted section.

$\therefore$ The distance between the centre of the sphere and the plane $= MN$

$$, \because MN = \frac{|2(-3) - (-2) - 2(1) + 12|}{\sqrt{4+1+4}} = 2 \text{ length unit.}$$

$\therefore$ The radius length of the circle $= \sqrt{15 - 4}$

$= \sqrt{11}$ length unit.

$\therefore$ Area of the resulted section $= 11\pi$ square unit.

5 ⓑ **6** ⓒ **7** ⓑ

8

$$\because T_9 = T_{10} \qquad \therefore \frac{T_{10}}{T_9} = \frac{20-9+1}{9} \times \frac{\frac{3}{x^2}}{2x} = 1$$

$\therefore \frac{2}{x^3} = 1 \qquad \therefore x^3 = 2 \qquad \therefore x = \sqrt[3]{2}$

Let the two terms be T_r and T_{r+1}

$$, \because \frac{T_{r+1}}{T_r} = \frac{15}{8} \qquad \therefore \frac{20-r+1}{r} \times \frac{\frac{3}{x^2}}{2x} = \frac{15}{8}$$

$, \therefore x = \sqrt[3]{2}$

$\therefore \frac{21-r}{r} \times \frac{3}{2 \times 2} = \frac{15}{8}$

$\therefore \frac{21-r}{r} = \frac{5}{2} \qquad \therefore 5r = 42 - 2r$

$\therefore 7r = 42 \qquad \therefore r = 6$

The two terms are T_6 and T_7

$$, \because T_{r+1} = {}^{20}C_r (2x)^{20-r}\left(\frac{3}{x^2}\right)^r = [{}^{20}C_r \times 2^{20-r} \times 3^r]\, x^{20-3r}$$

In case of existence of term free of x

$\therefore 20 - 3r = 0 \qquad x = \frac{20}{3} \notin \mathbb{Z}^+$

$\therefore$ There is no free term of x

9 ⓐ **10** ⓒ **11** ⓑ

12

[a] $z = 4 + 4\sqrt{3}\, i$

$\therefore r = \sqrt{(4)^2 + (4\sqrt{3})^2} = 8$

$\because x > 0, \ y > 0 \quad \therefore \theta$ lies in the first quadrant.

$\therefore \theta = \tan^{-1}\left(\frac{4\sqrt{3}}{4}\right) = 60° = \frac{\pi}{3}$

$\therefore z = 8\left(\cos\frac{\pi}{3} + i \sin\frac{\pi}{3}\right)$

$\therefore$ The exponential form for the number $z = 8\, e^{\frac{\pi}{3} i}$

$\therefore$ The square roots for the number z

$$= 2\sqrt{2}\, e^{\left(\frac{\frac{\pi}{3} + 2\pi n}{2}\right) i}$$

Where $n = 0, 1$

at $n = 0$:

The first square root $= 2\sqrt{2}\, e^{\frac{\pi}{6} i}$

at $n = 1$

The second square root $= 2\sqrt{2}\, e^{\frac{7}{6}\pi i} = 2\sqrt{2}\, e^{\frac{-5}{6}\pi i}$

[b] The expression $= \left[k - \frac{k-1}{-\omega^2} + (k+1)\,\omega^2\right]^8$

$= [k + \omega(k-1) + (k+1)\,\omega^2]^8$

$= [k + k\omega - \omega + k\omega^2 + \omega^2]^8$

$= [k(1 + \omega + \omega^2) + (\omega^2 - \omega)]^8$

$= [0 + (\pm\sqrt{3}\, i)]^8 = 81$

13 ⓑ

14

$(1,-1,3)\cdot\vec{r}=(1,-1,3)\cdot(-3,4,2)$

$\therefore (1,-1,3)\cdot\vec{r}=-1$ (Vector form)

$, 1(x+3)-(y-4)+3(z-2)=0$ (Standard form)

$, x-y+3z+1=0$ (General form)

15

$x+2y+3z=0$

$, 2x+3y+5z=0$

$, 3x-y+2z=0$

$$\therefore \begin{pmatrix}1&2&3\\2&3&5\\3&-1&2\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}0\\0\\0\end{pmatrix}$$

$$\therefore A=\begin{pmatrix}1&2&3\\2&3&5\\3&-1&2\end{pmatrix} \quad \therefore |A|=0$$

$$, \because \begin{vmatrix}1&2\\2&3\end{vmatrix}\neq 0$$

$\therefore RK(A)=RK(A^*)=2<$ the number of variables.

$\therefore$ The equations have an infinite number of solutions to find the form of solution we put $x=\ell$ by substituting in 1[st] equation

$\therefore \ell+2y+3z=0$ (1)

By substituting in 3[rd] equation

$\therefore 3\ell-y+2z=0$ (2)

Multiplying (2) by 2 and adding to (1) :

$\therefore 7\ell+7z=0 \qquad \therefore z=-\ell$

$,$ from (1) : $\therefore \ell+2y+3(-\ell)=0 \qquad \therefore y=\ell$

$\therefore$ The equations have an infinite number of solutions in the form $(\ell,\ell,-\ell)$

16 (c) **17** (c)

18

$$\text{L.H.S.}=\begin{vmatrix}x+a&x+a&2a\\a&x&a\\0&a-x&x-a\end{vmatrix}$$

(by doing $(R_1-R_2),(R_3+R_2)$)

$$=\begin{vmatrix}x&a&a\\a&x&a\\a&a&x\end{vmatrix} \text{ by doing } (C_1+C_2+C_3)$$

$$=\begin{vmatrix}x+2a&a&a\\x+2a&x&a\\x+2a&a&x\end{vmatrix}$$

(by taking $(x+2a)$ a common factor from C_1)

$$=(x+2a)\begin{vmatrix}1&a&a\\1&x&a\\1&a&x\end{vmatrix}$$

(by doing $(R_2-R_1),(R_3-R_1)$)

$$=(x+2a)\begin{vmatrix}1&a&a\\0&x-a&0\\0&0&x-a\end{vmatrix}$$

$=(x+2a)(x-a)^2=$ R.H.S.

19

At the intersection point : $\vec{r_1}=\vec{r_2}$

$\therefore (0,1,0)+t_1(1,2,-1)$

$=(1,1,1)+t_2(-2,-2,0)$

$t_1=1-2t_2$, then $t_1+2t_2=1$ (1)

$, 1+2t_1=1-2t_2$, then $t_1+t_2=0$ (2)

$, -t_1=1$, then $t_1=-1$ (3)

From (1) , (3) :

$\therefore t_2=1$

by substitute in equation (2) :

$\therefore t_1+t_2=-1+1=0$ (satisfying the equation).

$\therefore$ The two straight lines intersect at one point its coordinates is (x,y,z)

$=(0,1,0)+(-1)(1,2,-1)=(-1,-1,1)$

First session 2020

1 (b) **2** (c)

3

$$\text{[a] L.H.S.}=\begin{vmatrix}1&b&c\\b&1+b^2&bc\\c&bc&1+c^2\end{vmatrix}$$

by doing $(R_2-bR_1),(R_3-cR_1)$

$$=\begin{vmatrix}1&b&c\\0&1&0\\0&0&1\end{vmatrix}=1$$

4

$\vec{d_1}=(-2,0,2), \vec{d_2}=(0,3,-3)$

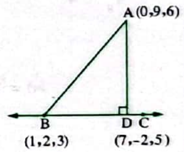
$$\cos\theta=\frac{|(-2,0,2)\cdot(0,3,-3)|}{\sqrt{(-2)^2+(2)^2}\sqrt{(3)^2+(-3)^2}}=\frac{|-6|}{2\sqrt{2}\times3\sqrt{2}}=\frac{1}{2}$$

$\therefore m(\angle\theta)=60°$

5 (b)

6

Find the equation of the straight line $\overleftrightarrow{BC}$ then the direction vector of the straight line is $\overrightarrow{BC}$



$=(7,-2,5)-(1,2,3)$

$=(6,-4,2)$

$\therefore$ Equation of the straight line $\overleftrightarrow{BC}$ is :

$\vec{r}=(1,2,3)+t(6,-4,2)$

$\therefore$ Let the point of projection is :

$D=(1+6t,2-4t,3+2t)$

$\therefore \overrightarrow{AD}=(1+6t,2-4t,3+2t)-(0,9,6)$

$=(1+6t,-7-4t,-3+2t)$

$\because \overrightarrow{AD}\perp\overrightarrow{BC} \quad \therefore \overrightarrow{AD}\cdot\overrightarrow{BC}=0$

$(1+6t,-7-4t,-3+2t)\cdot(6,-4,2)=0$

$6+36t+28+16t+(-6)+4t=0$

$\therefore 56t=-28$

$\therefore t=-\frac{1}{2}$ $\qquad \therefore$ The point $D=(-2,4,2)$

7

$$\text{L.H.S.}=\left(\frac{3+5\omega+3\omega^2}{1-2\omega-4\omega^2}+\frac{3+5\omega^2+3\omega}{1-2\omega^2-4\omega}\right)$$

$$=\frac{5\omega-3\omega}{1-2\omega-4(-1-\omega)}+\frac{5\omega^2-3\omega^2}{1-2\omega^2-4(-1-\omega^2)}$$

$$=\frac{2\omega}{1-2\omega+4+4\omega}+\frac{2\omega^2}{1-2\omega^2+4+4\omega^2}$$

$$=\frac{2\omega}{5+2\omega}+\frac{2\omega^2}{5+2\omega^2}$$

$$=\frac{2\omega(5+2\omega^2)+2\omega^2(5+2\omega)}{(5+2\omega)(5+2\omega^2)}$$

$$=\frac{10\omega+4+10\omega^2+4}{25+10\omega^2+10\omega+4}=\frac{8+10(-1)}{29+10(-1)}$$

$=\frac{-2}{19}=$ R.H.S.

8 (d) **9** (c) **10** (a)

11

[a] $z=1-\sqrt{3}\,i$

$, |z|=\sqrt{(1)^2+(-\sqrt{3})^2}=2$

$, \because x>0 , y<0$

$\therefore \theta$ lies in the 4[th] quadrant.

$, \theta=\tan^{-1}\left(\frac{-\sqrt{3}}{1}\right)=-60°=\frac{-\pi}{3}$

$\therefore z=2\left(\cos\frac{-\pi}{3}+i\sin\frac{-\pi}{3}\right)$

i.e. $z=2e^{-\frac{\pi}{3}i}$

The two square roots of z

$=\sqrt{2}\,e^{\left(\frac{-\frac{\pi}{3}+2\pi n}{2}\right)i}$ where $n=0,1$

at $n=0$ $\therefore$ The first square root $=\sqrt{2}\,e^{-\frac{\pi}{6}i}$

at $n=1$ $\therefore$ The second square root $=\sqrt{2}\,e^{\frac{5\pi}{6}i}$

[b] $z_1=\left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)^4=(\cos30°+i\sin30°)^4$

$=\cos120°+i\sin120°$

$, z_2=\sin60°+i\cos60°=\cos30°+i\sin30°$

$$\therefore z=\frac{z_1}{z_2}=\frac{\cos120°+i\sin120°}{\cos30°+i\sin30°}=\cos90°+i\sin90°$$

i.e. $z=\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}=e^{\frac{\pi}{2}i}$

12 (a) **13** (c) **14** (b)

15

[a] (1) $\because$ (45° , 60° , θ) are the direction angles of vector $\overrightarrow{A}$

$\therefore \cos^2 45° + \cos^2 60° + \cos^2 \theta = 1$

$\therefore \cos^2 \theta = \frac{1}{4}$

$\therefore \cos \theta = \pm \frac{1}{2}$ (negative refused)

$\therefore m(\angle \theta) = 60°$

(2) $\overrightarrow{A} = 10\left(\cos 45° , \cos 60° , \cos 60°\right)$

$= 10\left(\frac{1}{\sqrt{2}} , \frac{1}{2} , \frac{1}{2}\right) = 5\sqrt{2}\,\hat{i} + 5\,\hat{j} + 5\,\hat{k}$

[b] $\because \frac{2}{3} , \frac{-2}{3} , \frac{1}{3}$ are the direction cosines of $\overrightarrow{A}$

$\therefore \overrightarrow{A} = 6\left(\frac{2}{3} , \frac{-2}{3} , \frac{1}{3}\right) = (4 , -4 , 2)$

$, \overrightarrow{A} \times \overrightarrow{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -4 & 2 \\ -2 & 3 & 5 \end{vmatrix} = -26\,\hat{i} - 24\,\hat{j} + 4\,\hat{k}$

$\therefore$ The area of the parallelogram $= \|\overrightarrow{A} \times \overrightarrow{B}\|$

$= \sqrt{(-26)^2 + (-24)^2 + (4)^2} = 2\sqrt{317}$

$\simeq 35.6$ square unit.

16 (c)

17 (d)

18 (b)

19

[a] $\because T_{r+1} = {}^8C_r \left(-\frac{1}{x}\right)^r (x^3)^{8-r}$

$= {}^8C_r (-1)^r (x)^{-r+24-3r}$

$= {}^8C_r (-1)^r (x)^{24-4r}$

Put $24 - 4r = 0$

$\therefore r = 6$

$\therefore T_7$ is the term free of x

$, T_7 = {}^8C_6 (-1)^6 = 28$

[b] The middle term $= T_{\frac{8}{2}+1} = T_5$

$\therefore \frac{T_5}{T_6} = \frac{5}{8-5+1} \times \frac{x^3}{-\frac{1}{x}} = \frac{-5}{4} x^4$

at $x = 1$

$\therefore \frac{T_5}{T_6} = \frac{-5}{4}$

Second session 2020

1 (c) **2** (d) **3** (b)

4

[a] $\because \overrightarrow{C} = 5\sqrt{2}\,i + 5\,j + 5\,k$

$\therefore \|\overrightarrow{C}\| = \sqrt{\left(5\sqrt{2}\right)^2 + (5)^2 + (5)^2} = 10$

$\therefore \cos \theta_x = \frac{5\sqrt{2}}{10} = \frac{\sqrt{2}}{2}$ $\therefore \theta_x = 45°$

$, \cos \theta_y = \frac{5}{10} = \frac{1}{2}$ $\therefore \theta_y = 60°$

$, \cos \theta_z = \frac{5}{10} = \frac{1}{2}$ $\therefore \theta_z = 60°$

[b] $\sin \theta = \frac{\|\overrightarrow{A} \times \overrightarrow{B}\|}{\|\overrightarrow{A}\|\|\overrightarrow{B}\|} = \frac{65}{5 \times 26} = \frac{1}{2}$

$\therefore \theta = 30°$ or $150°$

5 (a) **6** (b) **7** (c)

8

(1) $\frac{T_6}{T_5} = \frac{8-5+1}{5} \times \frac{\frac{2}{x}}{x^2} = \frac{8}{5x^3}$

* If $\frac{T_6}{T_5} = \frac{25}{8}$

$\therefore \frac{8}{5x^3} = \frac{25}{8}$ $\therefore 125\,x^3 = 64$

$\therefore x^3 = \frac{64}{125}$ $\therefore x = \frac{4}{5}$

(2) $T_{r+1} = {}^8C_r \left(\frac{2}{x}\right)^r (x^2)^{8-r} = {}^8C_r (2)^r (x)^{-r} (x)^{16-2r}$

$= {}^8C_r (2)^r (x)^{16-3r}$

, put $16 - 3r = 0$ $\therefore r = \frac{16}{3} \notin \mathbb{Z}^+$

$\therefore$ There is no term free of x

9 (b) **10** (c)

11

by taking $(3\,x)$ common factor from R_1

$\therefore$ Determinant $= 3\,x \begin{vmatrix} 1 & 1 & 1 \\ 1 & b & a \\ a+b & a+1 & b+1 \end{vmatrix}$

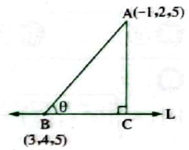
(by doing $(R_3 + R_2 + R_1)$)

$= 3\,x \begin{vmatrix} 1 & 1 & 1 \\ 1 & b & a \\ a+b+2 & a+b+2 & a+b+2 \end{vmatrix}$

by taking $(a + b + 2)$ common factor from $\mathbb{R}_3$

$= 3\,x\,(a+b+2) \begin{vmatrix} 1 & 1 & 1 \\ 1 & b & a \\ 1 & 1 & 1 \end{vmatrix} = 0$ (because $\mathbb{R}_1 = \mathbb{R}_3$)

12



Let the direction vector of the straight line $\overrightarrow{r}$ is $d_1 = (2 , -3 , 6)$

$, d_2 = \overrightarrow{AB} = (4 , 2 , 0)$

$\cos \theta = \frac{|(2 , -3 , 6) \cdot (4 , 2 , 0)|}{\sqrt{4+9+36}\sqrt{16+4}}$

$\therefore m(\angle \theta) \simeq 86° 20' , \|\overrightarrow{AB}\| = \sqrt{4^2 + 2^2} = \sqrt{20}$

$\therefore$ The length of the perpendicular (AC)

$= AB \sin \theta = \sqrt{20} \sin 86° 20' \simeq 4.46$ length unit

i.e. the distance between the point and the straight line = 4.46 length unit.

13 (d)

14

At the intersection point of the two straight lines :

$\overrightarrow{r_1} = \overrightarrow{r_2}$

$\therefore (0 , 1 , 0) + t_1 (1 , 2 , -1)$

$= (1 , 1 , 1) + t_2 (-2 , -2 , 0)$

$\therefore t_1 = 1 - 2t_2$ $\therefore t_1 + 2t_2 = 1$ (1)

$, 1 + 2t_1 = 1 - 2t_2$

$\therefore 2t_1 + 2t_2 = 0$ (2)

$, -t_1 = 1$ $\therefore t_1 = -1$ (3)

From (1) , (2) we get : $t_1 = -1 , t_2 = 1$

and this value , satisfy (3) :

$\therefore (x , y , z) = (0 , 1 , 0) + (-1)(1 , 2 , -1)$

$= (-1 , -1 , 1)$

$\therefore$ The intersection point is $(-1 , -1 , 1)$

15

$$\text{L.H.S.} = \left(\frac{1}{1+\omega i} - \frac{\omega + i}{1+\omega^2 i}\right)^8$$

$$= \left(\frac{1+\omega^2 i - (\omega + i + \omega^2 i - \omega)}{1+\omega^2 i + \omega i + (-1)}\right)^8$$

$$= \left(\frac{1-i}{\omega^2 i + \omega i}\right)^8 = \left(\frac{1-i}{i(-1)}\right)^8$$

$$= \left(\frac{1-i}{-i}\right)^8 = \left(\frac{(1-i)^2}{(-i)^2}\right)^4$$

$$= \left(\frac{-2i}{-1}\right)^4 = \frac{16}{1} = 16 = \text{R.H.S.}$$

16 (d) **17** (c) **18** (b)

19

[a] $r = \sqrt{(1)^2 + (\sqrt{3})^2} = 2$

$, \because x > 0 , y > 0$

$\therefore \theta$ lies in the first quadrant.

$\theta = \tan^{-1}(\sqrt{3}) = \frac{\pi}{3}$

$, z = 2\left(\cos\frac{\pi}{3} + i \sin\frac{\pi}{3}\right)$

i.e. $z = 2 e^{\frac{\pi}{3} i}$

The square roots of the number z

are $\sqrt{2}\, e^{\left(\frac{\frac{\pi}{3} + 2\pi n}{2}\right) i}$ where $n = 0 , 1$

at $n = 0$

$\therefore$ The first square root $= \sqrt{2}\, e^{\frac{\pi}{6} i}$

at $n = 1$

$\therefore$ The second square root $= \sqrt{2}\, e^{\frac{7\pi}{6} i} = \sqrt{2}\, e^{-\frac{5\pi}{6} i}$

[b] $z_1 = 1 - \sqrt{3}\, i$

$\therefore r = \sqrt{(1)^2 + (-\sqrt{3})^2} = 2$

$, \because x > 0 , y < 0$

$\therefore \theta$ lies in the fourth

$\therefore \theta = \tan^{-1}\left(\frac{-\sqrt{3}}{1}\right) = -60° = -\frac{\pi}{3}$

$\therefore z_1 = 2\left[\cos\left(-\frac{\pi}{3}\right) + i \sin\left(-\frac{\pi}{3}\right)\right]$

$, z_2 = e^{-\frac{\pi}{6} i} = \cos\left(-\frac{\pi}{6}\right) + i \sin\left(-\frac{\pi}{6}\right)$

$, z_3 = \left(\cos\left(-\frac{\pi}{12}\right) + i \sin\left(-\frac{\pi}{12}\right)\right)^2$

$= \cos\left(-\frac{\pi}{6}\right) + i \sin\left(-\frac{\pi}{6}\right)$

$$\therefore z = \frac{z_1 \cdot z_2}{z_3}$$

$$= \frac{2\left(\cos\frac{-\pi}{3} + i \sin\frac{-\pi}{3}\right)\left(\cos\frac{-\pi}{6} + i \sin\frac{-\pi}{6}\right)}{\cos\frac{-\pi}{6} + i \sin\frac{-\pi}{6}}$$

$$= 2\left[\cos\left(-\frac{\pi}{3}\right) + i \sin\left(-\frac{\pi}{3}\right)\right] = 2 e^{-\frac{\pi}{3} i}$$

First session 2021

1 (d)

Solution :

$\therefore 3 z_1 \times z_2 = 3(\cos\theta + i \sin\theta)(\cos 2\theta + i \sin 2\theta)$

$= 3(\cos 3\theta + i \sin 3\theta)$

$\therefore$ The principle amplitude of the complex number $(3 z_1 \times z_2)$ is 3θ

2 (b)

Solution :

$T_{r+1} = {}^{7n}C_r \left(\frac{-k}{x^2}\right)^r (x^5)^{7n-r} = {}^{7n}C_r (-k)^r x^{35n-7r}$

Put $35n - 7r = 0$ $\therefore r = 5n$

$\therefore$ The term free of x is T_{5n+1}

3 (c)

Solution :

$$\because 2 \times 2 \begin{vmatrix} a & b & c \\ 1 & -2 & 3 \\ e & f & d \end{vmatrix} = 2 \times 2 \times 8$$

$$\therefore \begin{vmatrix} 2a & 2b & 2c \\ 2 & -4 & 6 \\ e & f & d \end{vmatrix} = 32 \qquad \therefore \begin{vmatrix} 2 & -4 & 6 \\ 2a & 2b & 2c \\ e & f & d \end{vmatrix} = -32$$

4 (a)

Solution :

$\overrightarrow{CA} = \overrightarrow{CB} + \overrightarrow{BA} = (-\hat{j} + \hat{i} - \hat{k}) + (-2\hat{i} + 3\hat{j})$

$= -\hat{i} + 2\hat{j} - \hat{k}$

5 (a)

Solution :

$\because$ The two straight lines are perpendicular

$\therefore (-2 , m , 7) . (n , -4 , 2) = \text{zero}$

$\therefore -2n - 4m + 14 = 0$ $\therefore 2n + 4m = 14$

$\therefore n + 2m = 7$

6 (a)

Solution :

$\because \overrightarrow{n_1} = (2 , c , 4) , \overrightarrow{n_2} = (a + 2 , 6 , b - 2)$

$, \because$ the two planes are parallel

$\therefore \frac{2}{a+2} = \frac{c}{6} = \frac{4}{b-2}$ $\therefore 2(b-2) = 4(a+2)$

$\therefore 2b - 4 = 4a + 8$ $\therefore 4a - 2b = -12$

$\therefore 2a - b = -6$

7 (d)

Solution :

$\because \cos\theta_x = \frac{\overrightarrow{A}_x}{\|\overrightarrow{A}\|}$ $\therefore \frac{3}{13} = \frac{K}{\sqrt{K^2 + (12)^2 + (4)^2}}$

$\therefore 9(k^2 + 160) = 169 k^2$

$\therefore 9k^2 + 9 \times 160 = 169 k^2$

$\therefore 160 k^2 = 9 \times 160$ $\therefore k^2 = 9$

$\therefore k = -3$ (refused because the angle is acute)

or $k = 3$

8 (a)

Solution :

$$\therefore x - y = \frac{1}{1+\omega i} - \frac{\omega + i}{1 + \omega^2 i}$$

$$= \frac{(1+\omega^2 i) - (\omega + i)(1 + \omega i)}{(1+\omega i)(1+\omega^2 i)}$$

$$= \frac{1 + \omega^2 i - (\omega + i + \omega^2 i - \omega)}{1 + \omega i + \omega^2 i - 1}$$

$$= \frac{1-i}{(\omega + \omega^2) i} = \frac{1-i}{-i} = i + 1$$

9 (d)

Solution :

$\because$ The majority of the committee are women and the committee contains both genders

$\therefore$ The committee formed from (3 women and 2 men or 4 women and a man)

$= {}^6C_3 \times {}^9C_2 + {}^6C_4 \times {}^9C_1 = 855$

10 (b)

Solution :

$\because$ The coefficient of $T_6 = {}^{10}C_5$

$\therefore {}^{10}C_5 \times \left(\frac{1}{b}\right)^5 (a)^5 = {}^{10}C_5$

$\therefore \left(\frac{a}{b}\right)^5 = 1$ $\therefore \frac{a}{b} = 1$

11 (c)

Solution :

$\because (x-1)(x^2+x+1)=8 \qquad \therefore x^3-1=8$

$\therefore x^3=9$

$\therefore x^9+1=(9)^3+1=730$

12 (c)

Solution :

$\because -x\,y\,z=-100 \qquad (1)$

$, \frac{1}{2}\,x\,y\sin z=6.25 \qquad (2)$

Divide (1) by (2) : $\qquad \therefore \frac{z}{\sin z}=8$

$\therefore 2\,r=8$ cm

13 (d)

Solution :

$\because M_1M_2=\sqrt{(-3+2)^2+(2-1)^2+\left(-6\sqrt{2}+5\sqrt{2}\right)^2}$

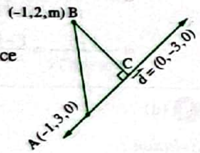
$=2$ length units

$, \because r_1-r_2=M_1M_2 \qquad \therefore 8-r_2=2$

$\therefore r_2=6$ length units.

14 (c)

Solution :



$\because$ The perpendicular distance

$(BC)=\frac{\|\overrightarrow{AB}\times\vec{d}\|}{\|\vec{d}\|}$

$\therefore \|\overrightarrow{AB}\times\vec{d}\|=8\times 3=24$

$, \because \overrightarrow{AB}=\vec{B}-\vec{A}=(0\,,-1\,,m)$

$\therefore \overrightarrow{AB}\times\vec{d}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 0 & -1 & m\\ 0 & -3 & 0\end{vmatrix}=3\,m\,\hat{i}$

$\therefore |3\,m|=24 \qquad \therefore 3\,m=24$

$\therefore m=8$ where $m\in\mathbb{R}^+$

15 (d)

Solution :

$\because \cos\theta=\frac{\|\overrightarrow{n_1}\cdot\overrightarrow{n_2}\|}{\|\overrightarrow{n_1}\|\|\overrightarrow{n_2}\|} \qquad \therefore \frac{1}{2}=\frac{|(1\,,1\,,0)\cdot(0\,,k\,,1)|}{\sqrt{(1)^2+(1)^2}\sqrt{k^2+(1)^2}}$

$\therefore 2|k|=\sqrt{2}\sqrt{k^2+1}$

$\therefore 4k^2=2(k^2+1) \qquad \therefore 2k^2=k^2+1 \qquad \therefore k^2=1$

$\therefore k=1$ (because $k>0$)

16 (c)

Solution :

$z=k\left(\sin\frac{4\pi}{3}-i\cos\frac{4\pi}{3}\right)$

$=k\left(\cos\frac{5\pi}{6}+i\sin\frac{5\pi}{6}\right)$

$\therefore z^6=k^6\left(\cos\frac{5\pi}{6}+i\sin\frac{5\pi}{6}\right)^6$

$=k^6(\cos 5\pi+i\sin 5\pi)=-k^6$

17 (a)

Solution :

$\because$ The two middle terms are equal

$\therefore a\,x=b \qquad \therefore x=\frac{b}{a}$

18 (b)

Solution :

$\because$ The point $(7\,,-2\,,2)$ lies on the surface of the sphere

$\therefore$ It satisfies its equation

$\therefore (7-4)^2+(-2-1)^2+(2+1)^2=k^2$

$\therefore k^2=27 \qquad \therefore |k|=3\sqrt{3}$

19 (a)

Solution :

Let $z=x+y\,i \qquad \therefore \bar{z}=x-y\,i$

$\therefore z+\bar{z}=2\,x \qquad \therefore 2\,x=2\,e^{\pi i}$

$\therefore x=-1$

$\therefore$ z could be any complex number has real part $=-1$

$\therefore$ z could be $e^{\pi i}$

20 (b)

Solution :

$\because \frac{{}^{n}C_4+{}^{n}C_3}{{}^{n+1}C_3}=\frac{{}^{n+1}C_4}{{}^{n+1}C_3}=\frac{(n+1)-4+1}{4}=\frac{n-2}{4}$

$\therefore \frac{n-2}{4}=1 \qquad \therefore n=6$

$\therefore \lfloor\underline{n-6}=\lfloor\underline{6-6}=1$

21 (a)

Solution :

$\because \left(x^2+2+\frac{1}{x^2}\right)^6=\left(\left(x+\frac{1}{x}\right)^2\right)^6=\left(x+\frac{1}{x}\right)^{12}$

$, \because T_{r+1}={}^{12}C_r\left(\frac{1}{x}\right)^r(x)^{12-r}={}^{12}C_r\,x^{12-2r}$

, put $12-2\,r=2 \qquad \therefore r=5$

$\therefore T_6$ is the term which contains x^2

$\therefore$ The coefficient of $T_6={}^{12}C_5$

22 (b)

Solution :

$\because a_{xy}=2\,x-y \qquad \therefore (A_{xy})=\begin{pmatrix}1 & 0 & -1\\ 3 & 2 & 1\\ 5 & 4 & 3\end{pmatrix}$

$, \because |A_{xy}|=\begin{vmatrix}1 & 0 & -1\\ 3 & 2 & 1\\ 5 & 4 & 3\end{vmatrix}=$ zero

$, \because \begin{vmatrix}1 & 0\\ 3 & 2\end{vmatrix}=2\neq$ zero

$\therefore$ RK $(A_{xy})=2$

23 (d)

Solution :

$\because \overrightarrow{DC}=\vec{C}-\vec{D}=(0\,,-4\,,6)$

$, \because \vec{A}\,/\!/\,\overrightarrow{DC}$

$\therefore \vec{A}=\sqrt{13}$ (the unit vector in $\overrightarrow{DC}$ direction)

$=\sqrt{13}\times\frac{(0\,,-4\,,6)}{\sqrt{(-4)^2+(6)^2}}=(0\,,-2\,,3)$

$\therefore \vec{A}\times\vec{B}=(0\,,-2\,,3)\times(-2\,,3\,,5)$

$=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 0 & -2 & 3\\ -2 & 3 & 5\end{vmatrix}=-19\,\hat{i}-6\,\hat{j}-4\,\hat{k}$

24 (c)

Solution :

$M=\left(\frac{1-2+1}{3}\,,\frac{2+0+4}{3}\,,\frac{4+5+0}{3}\right)=(0\,,2\,,3)$

$\therefore \overrightarrow{AM}$ is the direction vector of the straight line

$\overrightarrow{AM}=\vec{M}-\vec{A}=(-1\,,0\,,-1)$

$\therefore \vec{r}=(1\,,2\,,4)+t(-1\,,0\,,-1)$

25 (b)

Solution :

$\because \overrightarrow{AB}$ is a tangent to the circle

$\therefore (AB)^2=BC\times BD=BC\times(BC+CD)$

$=(BC)^2+BC\times CD$

$, \because$ the determinant $=(AB)^2+(BC)^2+CD\times BC$

$=(AB)^2+(AB)^2=2\,(AB)^2$

$\therefore 2\,(AB)^2=32 \qquad \therefore (AB)^2=16$

$\therefore AB=4$ length units

Second session 2021

1 ⓓ

Solution :

$T_{r+1} = {}^nC_r \left(\frac{5}{x}\right)^r (x^3)^{n-r}$

$= {}^nC_r\, 5^r\, x^{3n-4r}$

∵ The term free of x is T_7

∴ $r = 6$ ∴ $3n - 4(6) = 0$ ∴ $n = 8$

2 ⓑ

Solution :

∵ $\vec{A} + \overrightarrow{BC} = (4, 12, 9)$

∴ $(0, -1, 3) + \vec{C} - (4, -2, 1) = (4, 12, 9)$

∴ $\vec{C} + (-4, 1, 2) = (4, 12, 9)$

∴ $\vec{C} = (8, 11, 7)$

3 ⓓ

Solution :

∵ The vector perpendicular on the given plane is $(2, 3, -5)$

, ∵ the two planes are parallel.

∴ The vector perpendicular to the required plane is $(2, 3, -5)$

, ∵ the plane passes through the point $(-2, 2, -1)$

∴ Its equation :

$(2, 3, -5) \cdot \vec{r} = (2, 3, -5) \cdot (-2, 2, -1)$

∴ $2x + 3y - 5z = 7$

4 ⓒ

Solution :

$\frac{Z_1}{Z_2} = \frac{15\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)}{3(\cos\theta + i\sin\theta)}$

$= 5\left[\cos\left(\frac{\pi}{2} - \theta\right) + i\sin\left(\frac{\pi}{2} - \theta\right)\right]$

5 ⓐ

Solution :

(Take 7^n as a common factor from C_3)

∴ The determinant $= 7^n \begin{vmatrix} 7^{2n} & 7^{3n} & 7^{3n} \\ 7^{3n} & 7^{4n} & 7^{4n} \\ 7^{4n} & 7^{5n} & 7^{5n} \end{vmatrix}$

, ∵ $C_2 = C_3$

∴ Value of determinant = zero

6 ⓐ

Solution :

∵ The two straight lines are perpendicular

∴ $\vec{d_1} \cdot \vec{d_2}$ = zero

∴ $(-1, 3, 4) \cdot (m, n, 1) = 0$

∴ $-m + 3n + 4 = 0$

∴ $3n - m = -4$

7 ⓐ

Solution :

∵ $\vec{A} = (-2k, 2k, k)$

∴ $\vec{U_A} = \frac{(-2k, 2k, k)}{\sqrt{4k^2 + 4k^2 + k^2}} = \left(\frac{-2}{3}, \frac{2}{3}, \frac{1}{3}\right)$

∴ Cosine directions of the vector $\vec{A}$ is $\left(\frac{-2}{3}, \frac{2}{3}, \frac{1}{3}\right)$

8 ⓒ

9 ⓑ

Solution :

∵ Coefficient of T_{r+2} = Coefficient of T_{r+4}

∴ ${}^{20}C_{r+1} = {}^{20}C_{r+3}$

∴ $r + 1 = r + 3$ and hence $1 = 3$ (Refused)

or $r + 1 + r + 3 = 20$ ∴ $r = 8$

10 ⓑ

Solution :

The determinant $= \begin{vmatrix} 1 & \omega & \omega - 1 \\ 1 & -1 & \omega + 1 \\ 1 & \omega & \omega \end{vmatrix}$

By doing $(R_2 - R_1, R_3 - R_1)$

∴ The determinant $= \begin{vmatrix} 1 & \omega & \omega - 1 \\ 0 & -1-\omega & 2 \\ 0 & 0 & 1 \end{vmatrix}$

$= 1 \times (-1 - \omega) \times 1 = \omega^2$

11 ⓐ

Solution :

The straight line : $2x - 4 = \frac{2y - 8}{3} = \frac{2z - 14}{5}$ (Divide by 2)

∴ $L : \frac{x - 2}{1} = \frac{y - 4}{3} = \frac{z - 7}{5}$

∴ $\vec{d} = (1, 3, 5)$ and passes through the point $B(2, 4, 7)$ so the perpendicular distance = zero

12 ⓒ

Solution :

∵ $\vec{n_1} = (0, 0, 1)$, $\vec{n_2} = (1, 0, \sqrt{3})$

∴ $\cos\theta = \frac{|(0, 0, 1) \cdot (1, 0, \sqrt{3})|}{\sqrt{0 + 0 + 1}\sqrt{1 + 0 + 3}} = \frac{\sqrt{3}}{2}$

∴ $\theta = 30°$

13 ⓓ

Solution :

∵ D is the midpoint of $\overline{BC} = \left(\frac{2+0}{2}, \frac{3+3}{2}, \frac{7+1}{2}\right)$

$= (1, 3, 4)$

∴ The length of $\overline{AD} = \sqrt{(3-1)^2 + (1-3)^2 + (5-4)^2}$

$= \sqrt{4 + 4 + 1} = 3$ length units.

14 ⓐ

Solution :

The number of the straight linse passes through points from the first set $\{B, C, D, E\}$ and points from the second set $\{X, Y, Z, L, M\}$ added to the two straight lines carrying the two rays

$= 5 \times 4 + 2 = 22$ straight lines.

15 ⓑ

Solution :

∵ $\begin{vmatrix} a+2 & 3 & \sin C \\ 1 & b & 0 \\ 2 & 3 & \sin C \end{vmatrix} = 12$

Doing $(R_1 - R_3)$

∴ $\begin{vmatrix} a & 0 & 0 \\ 1 & b & 0 \\ 2 & 3 & \sin C \end{vmatrix} = 12$

∴ $ab\sin C = 12$

∴ The area of the $\Delta ABC = \frac{1}{2}ab\sin C$

$= 6$ square units.

16 ⓓ

Solution :

∵ Volume of the sphere $= \frac{4}{3}\pi r^3 = 36\pi$

∴ $r^3 = 27$ and hence $r = 3$ length units

∴ Equation of the sphere :

$(x + 1)^2 + y^2 + (z - 5)^2 = 9$

17 ⓐ

Solution :

∵ $5T_4, T_5, T_7, T_6$ are in proportion

∴ $\frac{5T_4}{T_5} = \frac{T_7}{T_6}$

∴ $5 \times \frac{4}{8 - 4 + 1} \times \frac{\sqrt{x}}{\left(\frac{1}{x}\right)} = \frac{8 - 6 + 1}{6} \times \frac{\left(\frac{1}{x}\right)}{\sqrt{x}}$

"multiply by $\frac{\sqrt{x}}{x}$"

∴ $4x = \frac{1}{2}\left(\frac{1}{x^2}\right)$ ∴ $x^3 = \frac{1}{8}$

∴ $x = \frac{1}{2}$

18 ⓓ

Solution :

$z_1 + z_2 = e^{5 + k\pi i} + e^{(5 + ki)\pi}$

$= e^{5 + k\pi i} + e^{5\pi + \pi k i}$

$= (e^5 + e^{5\pi})\, e^{k\pi i}$

, then the amplitude of $(z_1 + z_2)$ is $k\pi$

, ∵ $k \in \left]-\frac{1}{2}, \frac{1}{2}\right[$

∴ The amplitude $\in \left]\frac{-\pi}{2}, \frac{\pi}{2}\right[$

∴ The amplitude could be $\frac{-\pi}{6}$

19 ⓒ

Solution :

∵ $RK(A) = 3$ ∴ $|A| \neq$ zero

∴ $\begin{vmatrix} 1 & 2 & 3 \\ 2 & m & 6 \\ 5 & 7 & 9 \end{vmatrix} \neq$ zero

By doing $(R_2 - 2R_1)$, $(R_3 - 5R_1)$

∴ $\begin{vmatrix} 1 & 2 & 3 \\ 0 & m-4 & 0 \\ 0 & -3 & -6 \end{vmatrix} \neq$ zero by replacment

(C_2, C_3), then (R_2, R_3)

∴ $\begin{vmatrix} 1 & 3 & 2 \\ 0 & -6 & -3 \\ 0 & 0 & m-4 \end{vmatrix} \neq$ zero

∴ $-6(m - 4) \neq$ zero ∴ $m - 4 \neq$ zero

∴ $m \neq 4$

20 ⓐ

Solution :

The complex number is $\sqrt{k} - i\sqrt{k}$

$\therefore r = \sqrt{(\sqrt{k})^2 + (-\sqrt{k})^2} = \sqrt{2k}$

A lies in the fourth quadrant

, $\tan\theta = \frac{-\sqrt{k}}{\sqrt{k}} = -1$

$\therefore \theta = \frac{-\pi}{4}$

$\therefore$ The number is $\sqrt{2k}\, e^{\frac{-\pi}{4} i}$

21 ⓓ

Solution :

$\because T_{r+1} = {}^7C_r \left(\frac{a}{x^2}\right)^r (x)^{7-r}$

$= {}^7C_r\, a^r x^{7-3r}$

At $7 - 3r = 4$

$\therefore r = 1$

$\therefore$ Coefficient of $x^4 = {}^7C_1\, a^1 = 49$

$\therefore a = 7$

22 ⓓ

Solution :

$\because \overrightarrow{A} \times \overrightarrow{B} = \|\overrightarrow{A} \times \overrightarrow{B}\| . \overrightarrow{U}$

$= 5\left(\frac{3}{5}, 0, \frac{4}{5}\right) = (3, 0, 4)$

$\therefore (3\overrightarrow{A} + \overrightarrow{B}) \times (4\overrightarrow{A} + 2\overrightarrow{B})$

$= \overrightarrow{O} + 6\overrightarrow{A} \times \overrightarrow{B} + 4\overrightarrow{B} \times \overrightarrow{A} + \overrightarrow{O}$

$= 2\overrightarrow{A} \times \overrightarrow{B} = 2(3, 0, 4) = (6, 0, 8)$

23 ⓑ

Solution :

$\because (0, -1, 1)$ is a direction vector of the bisector of the angle between $\overleftrightarrow{Oy}$, $\overrightarrow{Oz}$

$\therefore$ Equation of the required straight line is

$\overrightarrow{r} = (2, -1, 4) + t(0, -1, 1)$

24 ⓒ

Solution :

$\because {}^nC_r : {}^{n-1}C_r = 3 : 1$

$\therefore \frac{n!}{(n-r)!\, r!} \times \frac{(n-r-1)!\, r!}{(n-1)!} = \frac{3}{1}$

$\therefore \frac{n\,(n-1)!}{(n-r)\,(n-r-1)!\, r!} \times \frac{(n-r-1)!\, r!}{(n-1)!} = \frac{3}{1}$

$\therefore \frac{n}{n-r} = \frac{3}{1}$

$\therefore 3n - 3r = n$

$\therefore 2n = 3r$

$\therefore \frac{n}{r} = \frac{3}{2}$

$\therefore \left(4\frac{n}{r}\right)! = 6! = 720$

25 ⓒ

Solution :

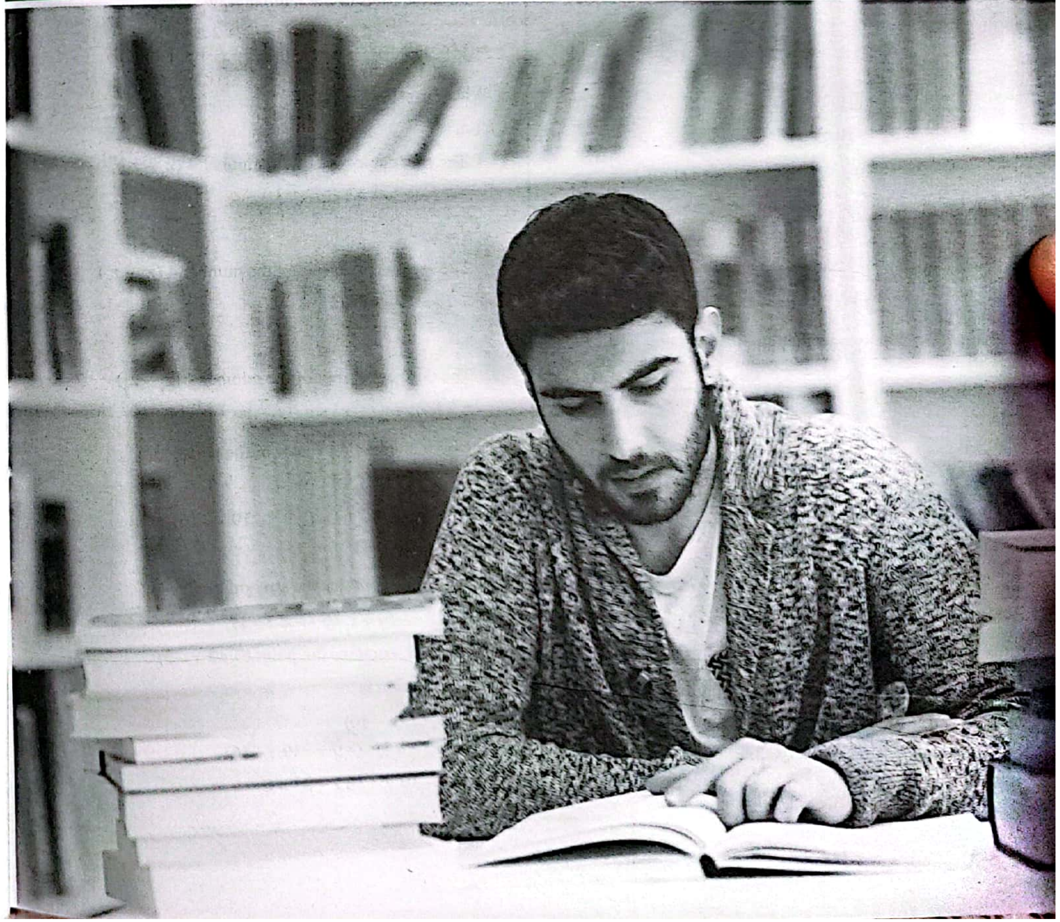
$\because a^2(-1-(-2)) - c^2(-1-0) + (-b^2)(1-0) = ac$

$\therefore a^2 + c^2 - b^2 = ac$

$\because \cos B = \frac{a^2 + c^2 - b^2}{2ac} = \frac{ac}{2ac} = \frac{1}{2}$

$\therefore m(\angle B) = 60°$

Answers of Al-Azhar Exams



Answers of Al-Azhar Exams

First session 2019

1 (1) (c) (2) (a) (3) (b) (4) (a) (5) (c) (6) (b)

2

[a] $\because \frac{{}^{n}C_{r}}{{}^{n}C_{r-1}} = \frac{7}{4}$ $\qquad \therefore \frac{n-r+1}{r} = \frac{7}{4}$

$\therefore 4n - 4r + 4 = 7r \qquad 4n - 11r = -4 \qquad (1)$

$, \frac{{}^{n}C_{r-1}}{{}^{n-1}C_{r-2} \times 5} = \frac{4}{6}$

$\therefore \frac{\lfloor n}{\lfloor n-r+1 \lfloor r-1} \times \frac{\lfloor n-r+1 \lfloor r-2}{5 \lfloor n-1} = \frac{2}{3}$

$\therefore \frac{n \lfloor n-1}{(r-1) \lfloor r-2} \times \frac{\lfloor r-2}{5 \lfloor n-1} = \frac{2}{3}$

$\therefore \frac{n}{5(r-1)} = \frac{2}{3} \qquad \therefore 3n = 10r - 10$

$\therefore 3n - 10r = -10 \qquad (2)$

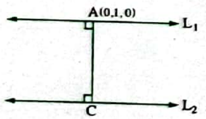
by solving the two equations (1) , (2) :

$\therefore n = 10 , r = 4$

[b] $\because \overrightarrow{d_1} = (1 , 2 , -1) , \overrightarrow{d_2} = (-2 , -4 , 2)$

$, \because \frac{1}{-2} = \frac{2}{-4} = \frac{-1}{2}$

$\therefore$ The two straight lines are parallel



let C be the projection drawn

From A to the straight line L_2

$\therefore C = (1 - 2t_2 , 1 - 4t_2 , 1 + 2t_2)$

$\therefore \overrightarrow{AC} = (1 - 2t_2 , -4t_2 , 1 + 2t_2)$

$\because \overrightarrow{AC} \perp L_2 \qquad \therefore \overrightarrow{AC} . \overrightarrow{d_2} = 0$

$\therefore (1 - 2t_2 , -4t_2 , 1 + 2t_2) . (-2 , -4 , 2) = 0$

$\therefore -2 + 4t_2 + 16t_2 + 2 + 4t_2 = 0$

$\therefore t_2 = 0 \qquad \therefore \overrightarrow{AC} = (1 , 0 , 1)$

$\therefore$ The length of the perpendicular

$= \| \overrightarrow{AC} \| = \sqrt{1 + 0 + 1} = \sqrt{2}$ length unit.

$\therefore$ The distance between the two straight lines

$= \sqrt{2}$ length unit.

3

[a] $z_1 = \frac{6 + 4i}{1 + i} \times \frac{1 - i}{1 - i} = 5 - i$

$z_2 = \frac{26}{5 - i} \times \frac{5 + i}{5 + i} = 5 + i$

$, z = 4(z_1 - z_2) = 4[(5 - i) - (5 + i)]$

$= -8i = 8e^{-\frac{\pi}{2} i}$

, the cubic roots of the number $4(z_1 - z_2)$

$= 2\left(\cos \frac{\frac{-\pi}{2} + 2\pi n}{3} + i \sin \frac{\frac{-\pi}{2} + 2\pi n}{3}\right)$

Where n $= 0 , 1 , 2$

at $n = 0$:

$\therefore$ The first cubic root of the number $4(z_1 - z_2)$

$= 2\left(\cos \frac{-\pi}{6} + i \sin \frac{-\pi}{6}\right)$

at $n = 1$:

$\therefore$ The second cubic root of the number $4(z_1 - z_2)$

$= 2\left(\cos \frac{\pi}{2} + i \sin \frac{\pi}{2}\right)$

at $n = 1$:

$\therefore$ The third cubic root of the number $4(z_1 - z_2)$

$= 2\left(\cos \frac{-5\pi}{6} + i \sin \frac{-5\pi}{6}\right)$

[b] The normal direction vector of the plane

$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 6 & -2 & -1 \\ 1 & -3 & 3 \end{vmatrix} = (-9 , -19 , -16)$

, $\because$ the plane contains the straight line

$\vec{r} = (0 , 3 , -5) + t_1 (6 , -2 , -1)$

$\therefore$ The plane contains the point $(0 , 3 , -5)$

$\therefore$ The equation is

$\vec{r} . (-9 , -19 , -16)$

$= (0 , 3 , -5) . (-9 , -19 , -16)$

$\therefore -9x - 19y - 16z - 23 = 0$

4

[a] L.H.S. $= \begin{vmatrix} a & -b & -c \\ b & 1 & 0 \\ c^2 & 0 & 1 \end{vmatrix}$ (by doing $c_1 - c^2 \times c_3$)

$= \begin{vmatrix} a + c^3 & -b & -c \\ b & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$ (by doing $c_1 - b\,c_2$)

$= \begin{vmatrix} a + b^2 + c^3 & -b & -c \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = a + b^2 + c^3$

[b] $\overrightarrow{A} . \overrightarrow{B} = 2\cos^2\theta + \log_3 x . \log_5 27 + 2\sin^2\theta = 11$

$\therefore \log_3 x . \log_5 27 = 9 \qquad \therefore \frac{\log x}{\log 3} . \frac{\log 27}{\log 5} = 9$

$\therefore \frac{\log x}{\log 3} . \frac{3 \log 3}{\log 5} = 9 \qquad \therefore \frac{\log x}{\log 5} = 3$

$\therefore \log x = 3 \log 5 \qquad \therefore \log x = \log 5^3$

$\therefore x = 125$

5

[a] The matrix equation :

$\begin{pmatrix} 1 & 1 & 2 \\ -2 & -1 & 1 \\ 1 & -1 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 5 \\ 6 \end{pmatrix}$

$\therefore A = \begin{pmatrix} 1 & 1 & 2 \\ -2 & -1 & 1 \\ 1 & -1 & 3 \end{pmatrix} \qquad \therefore |A| = 11$

$A^{-1} = \frac{1}{|A|} \times \text{adj}(A)$

$\therefore A^{-1} = \frac{1}{11} \begin{pmatrix} -2 & -5 & 3 \\ 7 & 1 & -5 \\ 3 & 2 & 1 \end{pmatrix}$

$\therefore \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{11} \begin{pmatrix} -2 & -5 & 3 \\ 7 & 1 & -5 \\ 3 & 2 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 5 \\ 6 \end{pmatrix} = \begin{pmatrix} -1 \\ -1 \\ 2 \end{pmatrix}$

$\therefore x = -1 , y = -1 , z = 2$

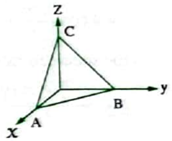
[b] $8x + 15y + 6z = 120$ (divide by 120)

$\therefore \frac{x}{15} + \frac{y}{8} + \frac{z}{20} = 1$

$\therefore A = (15 , 0 , 0)$

$, B = (0 , 8 , 0)$

$, C = (0 , 0 , 20)$



$\therefore \overrightarrow{AB} = (-15 , 8 , 0)$

$, \overrightarrow{AC} = (-15 , 0 , 20)$

$\therefore \overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -15 & 8 & 0 \\ -15 & 0 & 20 \end{vmatrix}$

$= 160\hat{i} + 300\hat{j} + 120\hat{k}$

$\therefore \| \overrightarrow{AB} \times \overrightarrow{AC} \| = \sqrt{(160)^2 + (300)^2 + (120)^2}$

≈ 360.56

$\therefore$ The area of $\Delta ABC = \frac{1}{2} \times \| \overrightarrow{AB} \times \overrightarrow{AC} \|$

≈ 180 square unit.

Second session 2019

1 (1) (b) (2) (a) (3) (b) (4) (c) (5) (d) (6) (a)

2

[a] $z = \frac{8}{1+\sqrt{3}\,i} \times \frac{1-\sqrt{3}\,i}{1-\sqrt{3}\,i} = 2 - 2\sqrt{3}\,i$

$, r = \sqrt{(2)^2 + \left(-2\sqrt{3}\right)^2} = 4$

$\because x > 0 \quad , \quad y < 0$

$\therefore \theta = \tan^{-1}\left(\frac{-2\sqrt{3}}{2}\right) = \frac{-\pi}{3}$

$\therefore$ The trigonometric form for z

$= 4\left(\cos\frac{-\pi}{3} + i \sin\frac{-\pi}{3}\right) = 4\,e^{-\frac{\pi}{3} i}$

, the square roots of the number z

$= 2\,e^{\left(\frac{-\frac{\pi}{3} + 2\pi n}{2}\right) i}$ where $n = 0 , 1$ at $n = 0$

$\therefore$ The first square root $= 2\,e^{-\frac{\pi}{6} i}$

, at $n = 1$

$\therefore$ The second square root $= 2\,e^{\frac{5\pi}{6} i}$

[b] The equation of the plane :

$x - 2y + 4z - 9 = 0$

The length of the perpendicular

$= \frac{|2 - 2(-1) + 4(4) - 9|}{\sqrt{1+4+16}} = \frac{11}{\sqrt{21}}$ length unit.

3

[a] $\because \frac{{}^{13}C_{r+1}}{{}^{13}C_r} = \frac{5}{9}$ $\qquad \therefore \frac{13-(r+1)+1}{r+1} = \frac{5}{9}$

$\therefore \frac{13-r}{r+1} = \frac{5}{9}$ $\qquad \therefore 5r + 5 = 117 - 9r$

$\therefore 14r = 112$ $\qquad \therefore r = 8$

$\therefore {}^{n}C_6 + {}^{n}C_7 = 3432$ $\qquad \therefore {}^{n+1}C_7 = 3432 = {}^{14}C_7$

$n + 1 = 14$ $\qquad \therefore n = 13$

[b] $\overrightarrow{d_1} = (2 , -1 , 1) \quad , \quad \overrightarrow{d_2} = (-2 , 7 , 11)$

$\therefore \overrightarrow{d_1} \cdot \overrightarrow{d_2} = (2 , -1 , 1) \cdot (-2 , 7 , 11)$

$= 2 \times -2 + (-1) \times 7 + 1 \times 11 =$ zero

$\therefore$ The two straight lines are perpendicular

$, \because$ at the point of intersection : $\overrightarrow{r_1} = \overrightarrow{r_2}$

$\therefore (1 , 2 , 4) + t_1 (2 , -1 , 1)$

$= (1 , 1 , 1) + t_2 (-2 , 7 , 11)$

$\therefore 1 + 2t_1 = 1 - 2t_2 \qquad \therefore t_1 + t_2 = 0 \qquad (1)$

$, 2 - t_1 = 1 + 7t_2 \qquad \therefore -t_1 - 7t_2 = -1$

$\therefore t_1 + 7t_2 = 1 \qquad (2)$

$, 4 + t_1 = 1 + 11t_2 \qquad \therefore t_1 - 11t_2 = -3 \qquad (3)$

from (1) , (2) we get : $t_1 = \frac{-1}{6} , t_2 = \frac{1}{6}$

by substitute in (3) : $\frac{-1}{6} - 11 \times \frac{1}{6} \neq -3$

i.e. These values don't satisfy equation (3)

$\therefore$ The two straight lines are not intersecting.

$\therefore$ The two straight lines are skew.

4

[a] $\because AX = B$

$$\therefore \begin{pmatrix} 2 & 1 & 3 \\ 1 & 1 & 0 \\ 0 & 1 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \\ 3 \end{pmatrix}$$

$$, A = \begin{pmatrix} 2 & 1 & 3 \\ 1 & 1 & 0 \\ 0 & 1 & 2 \end{pmatrix} \qquad \therefore |A| = 5$$

$, \because A^{-1} = \frac{1}{|A|} \times \text{adj}(A)$

$$\therefore A^{-1} = \frac{1}{5} \begin{pmatrix} 2 & 1 & -3 \\ -2 & 4 & 3 \\ 1 & -2 & 1 \end{pmatrix}$$

$$\therefore \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{5} \begin{pmatrix} 2 & 1 & -3 \\ -2 & 4 & 3 \\ 1 & -2 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ -1 \\ 3 \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}$$

$\therefore x = -2 \quad , \quad y = 1 \quad , \quad z = 1$

[b] $\overrightarrow{n_1} = (2 , 1 , -1) , \overrightarrow{n_2} = (3 , -2 , 0)$

$\therefore \cos\theta = \frac{|(2 , 1 , -1) \cdot (3 , -2 , 0)|}{\sqrt{4+1+1}\sqrt{9+4}} = \frac{4}{\sqrt{78}}$

$\therefore m(\angle\theta) \approx 63^\circ\, 4'$

5

[a] L.H.S. $= \begin{vmatrix} a & b & c \\ b & a & c \\ b & c & a \end{vmatrix}$ $\left(\text{by doing } (C_1 + C_2 + C_3)\right)$

(and taking $(a + b + c)$ a common factor from C_1)

$$= (a + b + c) \begin{vmatrix} 1 & b & c \\ 1 & a & c \\ 1 & c & a \end{vmatrix}$$

$\left(\text{by doing } (R_2 - R_1) , (R_3 - R_1)\right)$

$$= (a + b + c) \begin{vmatrix} 1 & b & c \\ 0 & a-b & 0 \\ 0 & c-b & a-c \end{vmatrix}$$

by interchanging R_2 , R_3

$$= -(a + b + c) \begin{vmatrix} 1 & b & c \\ 0 & c-b & a-c \\ 0 & a-b & 0 \end{vmatrix}$$

(by interchanging C_2 with C_3)

$$= (a + b + c) \begin{vmatrix} 1 & c & b \\ 0 & a-c & c-b \\ 0 & 0 & a-b \end{vmatrix}$$

$= (a + b + c)(a - c)(a - b) =$ R.H.S.

[b] We find the intersection points of x-axis with the sphere , the equation of x-axis $y = 0 , z = 0$

From the equation :

$(x - 2)^2 + 9 + 1 = 14 \qquad \therefore (x - 2)^2 = 4$

$x - 2 = \pm 2 \qquad \therefore x = 0$ or 4

$\therefore$ The points are $A(0 , 0 , 0) , B(4 , 0 , 0)$

$\therefore AB = 4$ length units.

First session 2020

1 (1) (c) (2) (c) (3) (c) (4) (b) (5) (b) (6) (c)

2

[a] Let the consecutive terms be T_r , T_{r+1} and T_{r+2}

$$\frac{\text{Coefficient of } T_{r+2}}{\text{Coefficient of } T_{r+1}} = \frac{7}{21} = \frac{1}{3}$$

$$\therefore \frac{n-(r+1)+1}{(r+1)} = \frac{1}{3} \quad \therefore 3n - 3r = r + 1$$

$$3n - 4r = 1 \qquad (1)$$

$$\frac{\text{Coefficient of } T_{r+1}}{\text{Coefficient of } T_r} = \frac{21}{35} = \frac{3}{5}$$

$$\frac{n-r+1}{r} = \frac{3}{5} \qquad \therefore 5n - 5r + 5 = 3r$$

$$5n - 8r = -5 \qquad (2)$$

From (1) and (2) : $n = 7$, $r = 5$

$\therefore$ The orders of the terms are T_5 , T_6 and T_7

[b] $\because \vec{A} \parallel \vec{B}$

$$\therefore \frac{1}{k} = \frac{6}{3} = \frac{2}{m}$$

$$\therefore k = \frac{1}{2} \ , \ m = 1$$

$$, \vec{C} = \left(\frac{1}{2} , 1 , \frac{3}{2}\right)$$

$$\|\vec{C}\| = \sqrt{\left(\frac{1}{2}\right)^2 + (1)^2 + \left(\frac{3}{2}\right)^2} = \frac{\sqrt{14}}{2}$$

3

[a] $\begin{vmatrix} x & 1 & 2 \\ 1 & x & 2 \\ 1 & 2 & x \end{vmatrix} = x - 1$ (by doing $C_1 + C_2 + C_3$)

$$\therefore \begin{vmatrix} x+3 & 1 & 2 \\ x+3 & x & 2 \\ x+3 & 2 & x \end{vmatrix} = x - 1$$

$\therefore (x+3) \begin{vmatrix} 1 & 1 & 2 \\ 1 & x & 2 \\ 1 & 2 & x \end{vmatrix} = x - 1$ (by doing $R_2 - R_1$, $R_3 - R_1$)

$\therefore (x+3) \begin{vmatrix} 1 & 1 & 2 \\ 0 & x-1 & 0 \\ 0 & 1 & x-2 \end{vmatrix} = x - 1$ (by interchanging C_2 with C_3)

$\therefore -(x+3) \begin{vmatrix} 1 & 2 & 1 \\ 0 & 0 & x-1 \\ 0 & x-2 & 1 \end{vmatrix} = x - 1$ (by interchanging R_2 with R_3)

$$\therefore (x+3) \begin{vmatrix} 1 & 2 & 1 \\ 0 & x-2 & 1 \\ 0 & 0 & x-1 \end{vmatrix} = x - 1$$

$$\therefore (x+3)(x-2)(x-1) = (x-1)$$

$$\therefore (x-1)\left[(x+3)(x-2) - 1\right] = \text{zero}$$

$$\therefore x = 1 \text{ or } x^2 + x - 6 - 1 = 0$$

$$\therefore x^2 + x - 7 = 0$$

$\therefore x = \frac{-1 \pm \sqrt{29}}{2}$ (refused)

$$\therefore S.S. = \{1\}$$

[b] Put $x = \frac{y-4}{-1} = \frac{z+1}{2} = t_2$

$$\therefore x = t_2 \ , y = -t_2 + 4 \ , z = 2t_2 - 1$$

at intersection of two straight lines

$$3 + 4t_1 = t_2$$

$$\therefore 4t_1 - t_2 = -3 \qquad (1)$$

$$, -1 + t_1 = -t_2 + 4 \quad \therefore t_1 + t_2 = 5 \qquad (2)$$

$$n + 3t_1 = 2t_2 - 1 \quad \therefore 3t_1 - 2t_2 = -1 - n \qquad (3)$$

From (1) , (2) : $t_1 = \frac{2}{5}$, $t_2 = \frac{23}{5}$

$\because$ The two straight lines are intersecting

$\therefore$ Values of t_1 , t_2 satisfy (3) :

$$\therefore 3 \times \frac{2}{5} - 2 \times \frac{23}{5} = -1 - n$$

$$\therefore n = 7$$

$\therefore$ The point of intersection is $\left(\frac{23}{5} , \frac{-3}{5} , \frac{41}{5}\right)$

4

[a] $z_1 = \left(\frac{\sqrt{3}}{2} + \frac{1}{2} i\right)^4 = \left(\cos\frac{\pi}{6} + i \sin\frac{\pi}{6}\right)^4$

$$= \left(\cos\frac{2\pi}{3} + i \sin\frac{2\pi}{3}\right)$$

$$, z_2 = \sin\frac{\pi}{3} + i\cos\frac{\pi}{3} = \cos\frac{\pi}{6} + i\sin\frac{\pi}{6}$$

$$\therefore z = \frac{\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}}{\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}} = \cos\frac{\pi}{2} + i\sin\frac{\pi}{2}$$

The square roots of the number z

$$= \cos\left(\frac{\frac{\pi}{2} + 2\pi n}{2} + i\sin\frac{\frac{\pi}{2} + 2\pi n}{2}\right)$$

Where $n = 0 , 1$

at $n = 0$:

$\therefore$ The first square root

$$= \cos\frac{\pi}{4} + i\sin\frac{\pi}{4}$$

at $n = 1$:

$\therefore$ The second square root

$$= \cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4} = \cos\frac{-3\pi}{4} + i\sin\frac{-3\pi}{4}$$

[b] $\overrightarrow{AB} = \vec{B} - \vec{A} = (-1 , 2 , 3) - (1 , 4 , 0)$

$$= (-2 , -2 , 3)$$

$$\|\overrightarrow{AB}\| = \sqrt{4 + 4 + 9} = \sqrt{17}$$

The component of $\vec{F}$ in direction of $\overrightarrow{AB}$

$$= \frac{\vec{F} \cdot \overrightarrow{AB}}{\|\overrightarrow{AB}\|} = \frac{(2 , -3 , 5) \cdot (-2 , -2 , 3)}{\sqrt{17}}$$

$$= \frac{-4 + 6 + 15}{\sqrt{17}} = \sqrt{17}$$

5

[a] L.H.S. $= \begin{vmatrix} y+z & x & x \\ y & z+x & y \\ z & z & x+y \end{vmatrix}$

$$= \begin{vmatrix} y & x & x \\ y & z+x & y \\ 0 & z & x+y \end{vmatrix} + \begin{vmatrix} z & x & x \\ 0 & z+x & y \\ z & z & x+y \end{vmatrix}$$

$$= y \begin{vmatrix} 1 & x & x \\ 1 & z+x & y \\ 0 & z & x+y \end{vmatrix} + z \begin{vmatrix} 1 & x & x \\ 0 & z+x & y \\ 1 & z & x+y \end{vmatrix}$$

by doing $(R_2 - R_1)$ in the 1st determinant

, $(R_3 - R_1)$ in the 2nd determinant.

$$= y \begin{vmatrix} 1 & x & x \\ 0 & z & y-x \\ 0 & z & x+y \end{vmatrix} + z \begin{vmatrix} 1 & x & x \\ 0 & z+x & y \\ 0 & z-x & y \end{vmatrix}$$

(by doing $(R_3 - R_2)$ in each of the two determinants)

$$= y \begin{vmatrix} 1 & x & x \\ 0 & z & y-x \\ 0 & 0 & 2x \end{vmatrix} + z \begin{vmatrix} 1 & x & x \\ 0 & z+x & y \\ 0 & -2x & 0 \end{vmatrix}$$

(by interchanging C_3 , C_2)

$$= y(1 \times z \times 2x) - z \begin{vmatrix} 1 & x & x \\ 0 & y & z+x \\ 0 & 0 & -2x \end{vmatrix}$$

$$= 2xyz - z(y)(-2x)$$

$$= 4xyz = R.H.S.$$

[b] $\overrightarrow{d_1} = (-2 , 0 , 2)$, $\overrightarrow{d_2} = (0 , 3 , -3)$

$$\cos\theta = \frac{|(-2 , 0 , 2) \cdot (0 , 3 , -3)|}{\sqrt{(-2)^2 + 2^2} \ \sqrt{3^2 + (-3)^2}}$$

$$= \frac{|-6|}{2\sqrt{2} \times 3\sqrt{2}} = \frac{1}{2}$$

$$\therefore m(\angle\theta) = 60^\circ$$

Second session 2020

1 (1) (b) (2) (a) (3) (d) (4) (b) (5) (d) (6) (b)

2

[a] $z^4 = 1 = \cos 0° + i \sin 0°$

$\therefore z = \left(\cos \frac{2\pi n}{4} + i \sin \frac{2\pi n}{4}\right)$

Where $n = 0, 1, 2, 3$

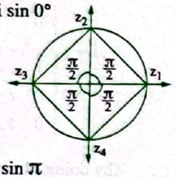
at $n = 0$: $\therefore z_1 = \cos 0° + i \sin 0°$

$\therefore z_1 = 1$

at $n = 1$:

$\therefore z_2 = \cos \frac{\pi}{2} + i \sin \frac{\pi}{2}$

$\therefore z_2 = i$



at $n = 2$: $\therefore z_3 = \cos \pi + i \sin \pi$

$\therefore z_3 = -1$

at $n = 3$:

$\therefore z_4 = \cos \frac{3\pi}{2} + i \sin \frac{3\pi}{2} = \cos \frac{-\pi}{2} + i \sin \frac{-\pi}{2}$

$\therefore z_4 = -i$

[b] Let $\theta_x = \theta_y = \theta_z = \theta$

$\cos^2\theta + \cos^2\theta + \cos^2\theta = 1$

$\therefore 3\cos^2\theta = 1$ $\quad \therefore \cos^2\theta = \frac{1}{3}$

$\therefore \cos\theta = \pm\frac{1}{\sqrt{3}}$

$\therefore \overrightarrow{A} = 7\sqrt{3}\left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right) = (7, 7, 7)$

or $\overrightarrow{A} = 7\sqrt{3}\left(\frac{-1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}\right) = (-7, -7, -7)$

3

[a] R.H.S. $= \begin{vmatrix} a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \\ 1 & 1 & 1 \end{vmatrix}$

by taking a, b, c a common factor from C_1, C_2, C_3 respectively

$= abc \begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ \frac{1}{a} & \frac{1}{b} & \frac{1}{c} \end{vmatrix} (R_3 \times abc)$

$= \begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ bc & ac & ab \end{vmatrix} =$ L.H.S.

[b] $\cos\theta = \frac{|(1, 1, 2) \cdot (\sqrt{3}-1, -\sqrt{3}-1, 4)|}{\sqrt{6} \times 2\sqrt{6}}$

$= \frac{|\sqrt{3}-1-\sqrt{3}-1+8|}{12} = \frac{1}{2}$

$\therefore m(\angle\theta) = 60°$

4

[a] (1) $T_{r+1} = {}^9C_r \left(\frac{-1}{2x^2}\right)^r (2x)^{9-r}$

$= {}^9C_r \left(-\frac{1}{2}\right)^r (2)^{9-r} x^{-2r+9-r}$

To find the coefficient of x^3

Put $-2r + 9 - r = 3$

$\therefore 3r = 6$ $\quad \therefore r = 2$

$\therefore$ The coefficient of $x^3 = {}^9C_2 \left(-\frac{1}{2}\right)^2 \times (2)^7$

$= 1152$

(2) The two middle terms are T_5, T_6

$\therefore T_5 = T_6$ $\quad \therefore \frac{T_6}{T_5} = 1$

$\therefore \frac{9-5+1}{5} \times \frac{\frac{1}{-2x^2}}{2x} = 1$

$\therefore \frac{-1}{4x^3} = 1$ $\quad \therefore x^3 = -\frac{1}{4}$

$\therefore x = \sqrt[3]{-\frac{1}{4}}$

[b] $\overrightarrow{A} \times \overrightarrow{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & 1 & 2 \\ 3 & 4 & -1 \end{vmatrix} = -9\hat{i} + 3\hat{j} - 15\hat{k}$

$\therefore$ Area of parallelogram $= \|\overrightarrow{A} \times \overrightarrow{B}\|$

$= \sqrt{(-9)^2 + (3)^2 + (-15)^2} = 3\sqrt{35}$ unit of area.

5

[a] Put $x = 1$ $\quad \therefore \Delta = 0$

$\begin{vmatrix} 1 & 2 & 2 \\ 1 & 1 & 1 \\ 3 & 4 & k \end{vmatrix} = 0$

$\therefore 1 \times (k-4) - 2(k-3) + 2(4-3) = 0$

$\therefore k = 4$

[b] Direction vector $= \overrightarrow{BC} = (2, 1, 0) - (-3, 2, 1)$

$= (5, -1, -1)$

The vector form of the straight line :

$\vec{r} = (1, -1, 0) + t(5, -1, -1)$

The parametric equations :

$x = 1 + 5t$, $y = -1 - t$, $z = -t$

The cartesian equation : $\frac{x-1}{5} = \frac{y+1}{-1} = \frac{z}{-1}$

$\because \frac{-14-1}{5} = -3$, $\frac{2+1}{-1} = -3$, $\frac{3}{-1} = -3$

$\therefore$ The point $(-14, 2, 3)$ lies on the straight line.

First session 2021

1 (1) (b) (2) (a) (3) (c) (4) (b) (5) (c) (6) (d)

2

[a] $z_1 = 1 + \sqrt{3}\,i$ $\qquad \therefore r_1 = \sqrt{1+3} = 2$

$, \because x > 0 , y > 0 \qquad \therefore \theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\sqrt{3}$

$\therefore \theta = \frac{\pi}{3} \qquad \therefore z_1 = 2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

$, z_2 = 1 + i \qquad \therefore r_2 = \sqrt{1+1} = \sqrt{2}$

$, \because x > 0 , y > 0 \qquad \therefore \theta = \tan^{-1}\frac{y}{x} = \tan^{-1} 1 = \frac{\pi}{4}$

$\therefore z_2 = \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

$\therefore z = \frac{z_1}{z_2} = \sqrt{2}\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)$

$\therefore z = \sqrt{2}\, e^{\frac{\pi}{12} i}$

The two square roots of the number z

$= \sqrt[4]{2}\, e^{\left(\frac{\frac{\pi}{12} + 2\pi r}{2}\right) i}$ where $r = 0 , 1$

$\therefore$ The first square root $= \sqrt[4]{2}\, e^{\frac{\pi}{24} i}$

, the second square root $= \sqrt[4]{2}\, e^{\frac{25\pi}{24} i}$

[b] $\because$ The sphere touches the plane

$\therefore$ The length of the perpendicular from the centre of the sphere to the plane = r

$\therefore r = \frac{|1+2+1-1|}{\sqrt{1+1+1}} = \sqrt{3}$ length unit

$\therefore$ The equation of the sphere is :

$(x-1)^2 + (y-2)^2 + (z-1)^2 = 3$

3

[a] $T_{r+1} = {}^9C_r\, (x^2)^{9-r} \left(\frac{1}{x}\right)^r = {}^9C_r\, x^{18-3r}$

put $18 - 3r = 0$ so $r = 6$

$\therefore$ The term free of $x = T_7 = {}^9C_6 = 84$

$\therefore \frac{T_7}{T_6} = \frac{9}{4}$

$\therefore \frac{9-6+1}{6} \times \frac{\left(\frac{1}{x}\right)}{x^2} = \frac{9}{4} \qquad \therefore \frac{2}{3x^3} = \frac{9}{4}$

$\therefore x^3 = \frac{8}{27} \qquad \therefore x = \frac{2}{3}$

[b] $\because \left(\sqrt{2} , 1 , -1\right)$ is the direction vector of the straight line

$, \left(\sqrt{2} , -1 , -1\right)$ is the direction vector perpendicular to the plane.

$\therefore \cos\theta = \frac{|\left(\sqrt{2} , 1 , -1\right) . \left(\sqrt{2} , -1 , -1\right)|}{\sqrt{2+1+1}\sqrt{2+1+1}} = \frac{2}{4} = \frac{1}{2}$

$\therefore m(\angle\theta) = 60°$

and this is the angle between the direction vector of the straight line and the perpendicular to the plane.

$\therefore$ Measure of the smaller angle between the straight line and the plane $= 90° - 60° = 30°$

4

[a] The matrix equation :

$$\begin{pmatrix} 1 & 1 & 2 \\ -2 & -1 & 1 \\ 1 & -1 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ 5 \\ 6 \end{pmatrix}$$

$$\therefore A = \begin{pmatrix} 1 & 1 & 2 \\ -2 & -1 & 1 \\ 1 & -1 & 3 \end{pmatrix} \qquad \therefore |A| = 11$$

$$\text{Adj}(A) = \begin{pmatrix} -2 & -5 & 3 \\ 7 & 1 & -5 \\ 3 & 2 & 1 \end{pmatrix}$$

$$\therefore A^{-1} = \frac{1}{11}\begin{pmatrix} -2 & -5 & 3 \\ 7 & 1 & -5 \\ 3 & 2 & 1 \end{pmatrix}$$

$$\therefore \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \frac{1}{11}\begin{pmatrix} -2 & -5 & 3 \\ 7 & 1 & -5 \\ 3 & 2 & 1 \end{pmatrix}\begin{pmatrix} 2 \\ 5 \\ 6 \end{pmatrix} = \begin{pmatrix} -1 \\ -1 \\ 2 \end{pmatrix}$$

$\therefore x = -1 \quad , \quad y = -1 \quad , \quad z = 2$

[b] The direction vectors $= (3 , 1 , -3)$

The vector equation :

$\vec{r} = (2 , 1 , -3) + t\,(3 , 1 , -3)$

, the parametric equation :

$x = 2 + 3t \quad , \quad y = 1 + t \quad , \quad z = -3 - 3t$

, the cartesian equation :

$\frac{x-2}{3} = \frac{y-1}{1} = \frac{z+3}{-3}$

5

[a] The L.H.S. $= \begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix}$

By doing $(R_2 - R_1 , R_3 - R_1)$

$$= \begin{vmatrix} 1 & a & bc \\ 0 & b-a & c(a-b) \\ 0 & c-a & b(a-c) \end{vmatrix}$$

$$= (a-b)(c-a)\begin{vmatrix} 1 & a & bc \\ 0 & -1 & c \\ 0 & 1 & -b \end{vmatrix}$$

By doing $(r_3 + r_2)$

$$= (a-b)(c-a)\begin{vmatrix} 1 & a & bc \\ 0 & -1 & c \\ 0 & 0 & c-b \end{vmatrix}$$

$= (a-b)(b-c)(c-a)$

[b] $3\cos^2\theta = 1$

$\therefore \cos^2\theta = \frac{1}{3}$

$\therefore \cos\theta = \pm\frac{1}{\sqrt{3}}$

$\therefore \vec{A} = 7\sqrt{3}\left(\frac{1}{\sqrt{3}} , \frac{1}{\sqrt{3}} , \frac{1}{\sqrt{3}}\right) = (7 , 7 , 7)$

or $\vec{A} = 7\sqrt{3}\left(\frac{-1}{\sqrt{3}} , \frac{-1}{\sqrt{3}} , \frac{-1}{\sqrt{3}}\right) = (-7 , -7 , -7)$

Second session 2021

1 (1) (d) (2) (c) (3) (b) (4) (a) (5) (b) (6) (c)

2

[a] The matrix equation is :

$$\begin{pmatrix} 2 & -3 & -1 \\ 1 & 2 & 3 \\ 1 & 0 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 9 \\ 15 \\ 12 \end{pmatrix}$$

$$\text{put } A = \begin{pmatrix} 2 & -3 & -1 \\ 1 & 2 & 3 \\ 1 & 0 & -2 \end{pmatrix} \qquad \therefore |A| = -21$$

$$A^{adj} = \begin{pmatrix} -4 & -6 & -7 \\ 5 & -3 & -7 \\ -2 & -3 & 7 \end{pmatrix}$$

$$\therefore A^{-1} = \frac{1}{-21}\begin{pmatrix} -4 & -6 & -7 \\ 5 & -3 & -7 \\ -2 & -3 & 7 \end{pmatrix}$$

$$\therefore \begin{pmatrix} x \\ y \\ z \end{pmatrix} = A^{-1} \times \begin{pmatrix} 9 \\ 15 \\ 12 \end{pmatrix}$$

$$= \frac{-1}{21}\begin{pmatrix} -4 & -6 & -7 \\ 5 & -3 & -7 \\ -2 & -3 & 7 \end{pmatrix} \times \begin{pmatrix} 9 \\ 15 \\ 12 \end{pmatrix} = \begin{pmatrix} 10 \\ 4 \\ -1 \end{pmatrix}$$

$\therefore x = 10 \quad , \quad y = 4 \quad , \quad z = -1$

[b] $(4 , 10 , -7) . \vec{r} = (4 , 10 , -7) . (2 , -1 , 0)$

$\therefore (4 , 10 , -7) . \vec{r} = -2$ (The vector form)

$4(x-2) + 10(y+1) - 7z = 0$

(The standard form)

$4x + 10y - 7z = -2$

$\therefore 4x + 10y - 7z + 2 = 0$ (The general form)

3

[a] $z = \frac{-\sqrt{3} - i}{\sqrt{3} - i} \times \frac{\sqrt{3} + i}{\sqrt{3} + i} = -\frac{1}{2} - \frac{\sqrt{3}}{2} i$

$\therefore$ The algebraic form of the number is

$z=-\frac{1}{2}-\frac{\sqrt{3}}{2}i$

$\therefore r=\sqrt{\left(-\frac{1}{2}\right)^2+\left(\frac{-\sqrt{3}}{2}\right)^2}=1$

$\because x<0 , y<0$

$\therefore \theta$ lies in the third quadrant.

$\therefore \theta=-\pi+\tan^{-1}\left(\frac{y}{x}\right)=-\pi+\frac{\pi}{3}=\frac{-2\pi}{3}$

The trigonometric form of the number is

$z=\cos\frac{-2\pi}{3}+i\sin\frac{-2\pi}{3}$

The exponential form of the number $z=e^{\frac{-2\pi}{3}i}$

The square roots of the number is

$z=e^{\left(\frac{\frac{-2\pi}{3}+2\pi n}{2}\right)i}$ where $n=0 , 1$

at $n=0$:

$\therefore$ The first square root of the number $=e^{\frac{-\pi}{3}i}$

at $n=1$:

The second square root of the number $=e^{\frac{2\pi}{3}i}$

[b] Divide by 4 :

$\therefore \frac{x}{4}+\frac{y}{4}+\frac{z}{4}=1$

$\therefore A(4 , 0 , 0) , B(0 , 4 , 0) , C(0 , 0 , 4)$

$\therefore AB=\sqrt{16+16+0}=4\sqrt{2}$ length units

$, BC=\sqrt{0+16+16}=4\sqrt{2}$ length units

$, AC=\sqrt{16+0+16}=4\sqrt{2}$ length units

$\therefore$ The perimeter of $\Delta ABC=4\sqrt{2}\times 3$

$=12\sqrt{2}$ length units.

4

[a] The order of the middle term $=\frac{8}{2}+1=5$

$\therefore T_5$ is the middle term

$, \because T_{r+1}={}^8C_r\times\left(\frac{1}{ax}\right)^r\times(x^2)^{8-r}$

$={}^8C_r\times a^{-r}\times x^{16-3r}$

put $16-3r=10$ $\quad\therefore r=2$

$\therefore T_3$ is the term includes x^{10}

$, \because$ the coefficient of T_5 = the coefficient of T_3

$\therefore {}^8C_4\times a^{-4}={}^8C_2\times a^{-2}$

$\therefore a^2=\frac{5}{2}$

$\therefore a=\pm\frac{\sqrt{10}}{2}$

[b] Put $y=0 , z=0$

$\therefore (x-2)^2+9+1=14$ $\quad\therefore (x-2)^2=4$

$\therefore x-2=\pm 2$ $\quad\therefore x=0 , x=4$

$\therefore$ The two points A , B are $(0 , 0 , 0) , (4 , 0 , 0)$

$\therefore$ The length of $\overline{AB}=4$ length units.

5

[a] L.H.S. $=\begin{vmatrix} x+a & x+a & 2a \\ a & x & a \\ 0 & a-x & x-a \end{vmatrix}$

(by doing $(R_1-R_2) , (R_3+R_2)$) .

$=\begin{vmatrix} x & a & a \\ a & x & a \\ a & a & x \end{vmatrix}$ by doing $(C_1+C_2+C_3)$

$=\begin{vmatrix} x+2a & a & a \\ x+2a & x & a \\ x+2a & a & x \end{vmatrix}$

(by taking $(x+2a)$ a common factor from C_1)

$=(x+2a)\begin{vmatrix} 1 & a & a \\ 1 & x & a \\ 1 & a & x \end{vmatrix}$

(by doing $(R_2-R_1) , (R_3-R_1)$)

$=(x+2a)\begin{vmatrix} 1 & a & a \\ 0 & x-a & 0 \\ 0 & 0 & x-a \end{vmatrix}$

$=(x+2a)(x-a)^2=$ R.H.S.

[b] $\overrightarrow{\theta_1}=(2 , 0 , -2) , \overrightarrow{\theta_2}=(1 , 2 , -2)$

$\cos\theta=\frac{|(2 , 0 , -2).(1 , 2 , -2)|}{\sqrt{4+0+4}\sqrt{1+4+4}}$

$=\frac{|2+0+4|}{2\sqrt{2}\times 3}=\frac{1}{\sqrt{2}}$

$\therefore \theta=45^\circ$

Notes

Notes

Notes

Notes

Notes

Notes

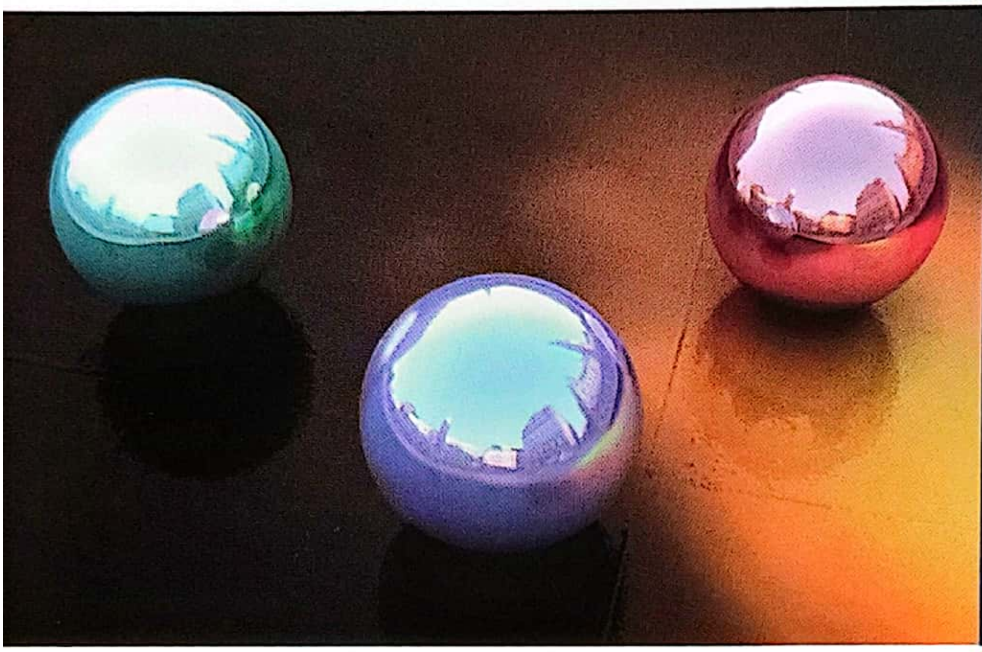
Notes

Notes

Notes

الجــزء الخـــاص بالإجـــابات يُصـــرف مجـــانًا مع الكـــتاب

Question Bank
& Practice Exams



3rd Sec
2022

AL TALADA BOOKSTORE
For printing, publication & distribution
El Faggala - Cairo - Egypt
Tel.: 02/ 259 340 12 - 259 377 91
e-mail: info@elmoasserbooks.com
www.elmoasserbooks.com
15014

/ElMoasser.eg

6 223007 310949